Semipaesti plura?

Franklin Wells
Knox College
Dec. 1850

J.S. Gaylord

Holl

Harwood

SOLAR AND LUNAR

ECLIPSES

FAMILIARLY ILLUSTRATED AND EXPLAINED,

WITH THE

METHOD OF CALCULATING THEM

ACCORDING TO THE

THEORY OF ASTRONOMY,

AS TAUGHT IN

NEW ENGLAND COLLEGES.

BY JAMES H. COFFIN, A. M.

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PREFACE.

The design of this treatise, is to explain the "*rationale*" of some of the most interesting astronomical calculations, in such a way that the student may clearly see the reason of every step, and its connection with the theory. In this respect it differs from many others, which give the *rules* for calculating merely, without any explanation of the reason of them. Being partly designed as a text book for colleges, the author has endeavoured to adapt it to the design of college education, which is not so much to make adept practitioners in any particular science, as to give broad and comprehensive views of the whole field. Hence, the *principles* of the several sciences should be thoroughly understood by the student; but the application of them to practice *by mere rules* is foreign to the design of a collegiate course of study. If, therefore, the calculations of astronomy are attended to at all in college, it should be in such a way, that the connection with the theory may be apparent, and that the two may mutually illustrate each other. In many of treatises for colleges, this point seems to be overlooked. Some of them contain tables for astronomical calculations which are very minute and accurate, and at the same time, so constructed and arranged as to reduce the labour of calculation as much as possible; but the student can see no connection between them and the motions and perturbations which occupy his attention in the study of the theory. In fact, one who has studied the theory with ever so much thoroughness, has here very little advantage over one who is entirely ignorant of it; each being guided wholly by rules that must appear entirely arbitrary.

Such tables not being adapted to the design of this treatise, the author found it necessary to prepare a set differing somewhat in

their construction from those in ordinary use. They are based, for the most part, on those of Delambre and Burg, with the more recent improvements of Airy and Bessel; but varied in the plan of construction so as to adapt them to this work. Calculations may be made from them sufficiently accurate for the ordinary purposes of an almanac; yet, as they are not designed especially for that purpose, the aim has been not so much to secure extreme accuracy in the results, as to render them easily understood.

The quantities in the tables are given, for the most part, in degrees and decimals, instead of signs, degrees, minutes, and seconds, with a view to facilitate the labour of calculation, and to secure the same degree of accuracy with a less number of figures.

The work is intended to be complete in itself, on the subject on which it treats, for those who have a general knowledge of the motions of the heavenly bodies; yet, it would, doubtless, contribute to a better understanding of it, to read some such work as Olmsted's or Herschell's Astronomy previously.

Williams College, October, 1843.

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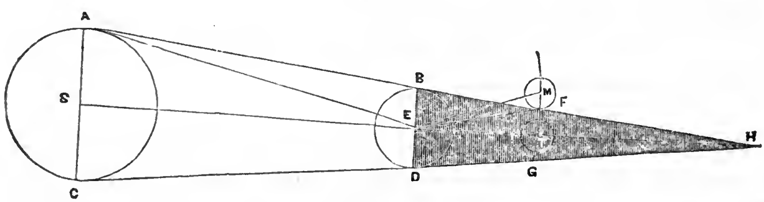
earth. The angle MES, is not difficult of computation, being equal to SEA+MEC+AEC, the latter of which is equal (Euc. 1, 32) to ECB—EAB. Hence, MES=SEA+MEC+ECB—EAB, all of which are easily found. The two former are the apparent semidiameters of the sun and moon, as viewed from the earth, and the two latter the semidiameter of the earth, as viewed from the moon and sun, or, respectively, the moon's and sun's horizontal parallax. On account of the distance of the sun and moon from the earth not being constant, the angle MES is subject to a variation in size, being sometimes 1° 38′, and at other times not more than 1° 14′. The reader will perceive that, when the sun and moon are in conjunction, the angle MES is the moon's latitude; and the conclusion to which we have just arrived, may be expressed thus: If the latitude of the moon, when new, is less than 1° 38′ there *may* be an eclipse of the sun, and if it is less than 1° 14′ there *must* be one.

Fig. 2.

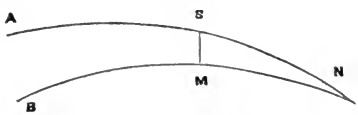


4. Again, let S (Fig. 2) represent the centre of the sun, E that of the earth, and M that of the moon, just impinging upon the earth's shadow. The angle MET, as represented in the figure, is plainly the least possible, without producing a partial or total eclipse of the moon. The angle MET=MEF+FET, both of which can be easily computed. The former is the moon's apparent simidiameter, as seen from the earth, and FET is that of the section of the earth's shadow that eclipses the moon. Now, (Euc. 1, 32,) FET=BFE—FHE and FHE=AES—BAE; therefore, FET=BFE+BAE—AES, the two former of which are the lunar and solar parallaxes, and the latter is the sun's apparent semidiameter, as seen from the earth. That is, *the apparent semidiameter of the section of the earth's shadow that eclipses the moon, is equal to the sum of the parallaxes of the sun and moon, diminished by the sun's apparent semidiameter.* And if this angle be increased by the moon's apparent semidiameter, MEF, we shall have the whole angle MET, which is the least latitude the moon can have in oppo-

sition, without being eclipsed. This angle varies in size, like the analagous one in solar eclipses, just described, and for the same reason. Its maximum value is 1° 4′, and its minimum 50′.

5. The centre of the section of the earth's shadow that eclipses the moon, must be situated in the plane of the ecliptic, directly opposite to the sun, and must, therefore, be at the same distance from one of the moon's nodes that the sun is from the other. Now, we wish to know how near the sun, in its annual course, may approach to one of the moon's nodes, without occasioning eclipses; or, in other words, at what distance from the node the moon's track and the ecliptic will have diverged, so as to be from 1° 14′ to 1° 38′ apart, if our inquiry relates to solar eclipses—or, from 50′ to 1° 4′, if it relates to lunar.

Fig. 3.



6. Let AN (Fig. 3) represent a portion of the ecliptic, BN a portion of the moon's orbit, SM a portion of a secondary to the ecliptic, and N one of the moon's nodes. Then, in the right angled spherical triangle, SMN, we have the angle SNM=5° 7′ 47″.9,* and for solar eclipses, the arc SM=1° 14′ to 1° 38′. With these data, we find NS to be from 13° 14′ to 19° 42′, according to the value we give to SM. Hence, if the sun is within 19° 42′ of the moon's node, on either side, at the time of new moon, it *may* be eclipsed; and if it is within 13° 24′, it *must* be. These distances are called the *solar ecliptic limits.*

Since it takes the sun more than a lunar month, usually, to pass over one of these arcs, it follows, that it must be eclipsed at every passage, and, consequently, twice a year, at least. It may be eclipsed twice during one passage; once, just after it enters the ecliptic limits, and again, just before it leaves them: but, if so, both of the eclipses will be small, and not central upon any part of the earth.

7. For lunar eclipses, the arc SM is 50′ to 1° 4′, and, by the same process as above, we find the *lunar ecliptic limits* to be, from 7° 47″ to 13° 21′ on each side of the node.† They embrace an

* This angle is subject to a slight variation, amounting, at its maximum, to 8′ 47″.15.
† Baily.

arc considerably less than is passed over by the sun in one lunation, so that it often happens, that the sun passes the node without there being any lunar eclipse.

The lunar ecliptic limits being so much less than the solar, eclipses of the moon must be proportionably less frequent; yet, since a lunar eclipse is always visible over half the earth's surface, while one of the sun can be seen only over a very much smaller section, there will, on an average, be a greater number of *visible* eclipses of the moon, at any given place, than of the sun.

8. As the moon's nodes are 180° apart, or, in opposite points of the ecliptic, the interval between eclipses occurring at one node, and those occurring at the other, must be about six months. Also, since the nodes move backward about 19° in each year, eclipses must happen, on an average, nearly three weeks earlier every year than they did on the year preceding. The reader will see that these conclusions are verified by past experience, if he will take the trouble to examine the almanacs of former years.

CHAPTER II.

MEAN TIME OF AN ECLIPSE, AND THE MEAN LONGITUDES AND ANOMALIES OF THE SUN AND MOON.

9. The sun, in its apparent annual course, leaves the vernal equinox where its longitude is 0°, about the 21st of March, and moves eastward, towards the moon's nodes, about 1° each day. Consequently, it must arrive at either node, in about as many days after the 21st of March, in any given year, as the longitude of the node, in that year, contains degrees. At the next new moon before or after the date thus found, (more frequently the former,) there will be an eclipse of the sun. It is probable, though not certain, that there will also be a lunar eclipse at the nearest full moon. It will occur then if at all at that passage of the node.

10. The velocity of the motions of sun and moon in their respective orbits, is variable; but it is more convenient in astronomical calculations to regard it as uniform, and to make the necessary corrections for the inequalities afterward.

11. To explain the method of calculating the time of an eclipse, we will take, as an example, the solar eclipse that will occur when the sun passes the moon's ascending node, in the year 1854. Table 2d, at the end of this volume, contains the time of new moon in March of that year, as well as of every other during the present century, and the longitude of the sun, moon, and moon's ascending node, all calculated on the supposition of a uniform rate of motion. It also gives the mean anomolies of the sun and moon, i. e., the distance of each from its perigee. The longitude of the descending node may be found by adding, or subtracting, 180° to, or from, that of the ascending node. We might compute all these quantities from the data given in table 1st, but table 2d supercedes the necessity. Although some of them will not be used in this chapter, it is most convenient to take them all out together, and write them as below. The longitude of the node on that year (see right hand column of the table) is 64°.2158 ; consequently, the sun will arrive at it about 64 days after the 21st of March, which carries the time to May 24th. The eclipse in question will occur at the new moon nearest that time. Entering table 3d,* we next take out such a number of lunations (in this case, two) as, when added to the time of new moon in March, will bring it near to the time when the sun reaches the moon's node, and write it down, with the longitudes and anomalies, under the corresponding quantities already taken from table 2d. These must be added together, (with the exception of the right hand column, where the lower number must be subtracted from the upper, because the motion of the node is retrograde,) and we thus obtain the time of mean new moon in May. The following shows the operation :—

	Time.	Sun's Anomaly.	Sun's Longitude.	Moon's Anomaly.	Moon's Longitude.	Longitude of Node.
	d. h. m. s.	°	°	°	°	°
Mean new moon in March,...	28 10 42 50	85.586	6.0194	93.665	6.0194	64.2158
Add two lunations,............	59 1 28 6	58.211	58.2135	51.634	58.2135	-3.1275
Mean new moon in May,.....	26 12 10 56	143.797	64.2329	145.299	64.2329	61.0883

Table 5th shows the month and day to which any number of days found by the foregoing addition corresponds.

At this stage of the calculation, it is well to compare the longitude of the sun and moon with that of the node, and if they do not

* Table 3d shows the length of any number of mean lunations, from one to thirteen, with the mean motions of the sun and moon, both in longitude and anomaly, during the same ; also, the mean motion of the moon's nodes.

differ more than 20°, there *may* be an eclipse, though there will not, *probably*, be one, if the difference is over $16\frac{1}{2}$°. If the difference is too great, it shows that too many lunations were added, or too few, and a correction must be made accordingly. A difference of over 11° shows that another eclipse, at that passage of the node, is possible, but not probable, unless it is as much as 14°.

12. At the time of new moon, the longitudes of the sun and moon must be equal, and, according to our calculations, they are so at the time of the mean new moon in May just found. But this is on the supposition, that their motions were uniform. To find whether or not their longitudes are truly equal, we shall proceed, in the following chapters, to compute them, taking into account all the chief inequalities in their motions; and, if they come out alike, the time of new moon is correctly found; otherwise, we shall have to add or subtract such an amount of time, as, with the relative velocities of the sun and moon, at the time, will render them equal. As a preparatory step, it is necessary to know, more accurately, the the moon's anomaly, and the longitude of its node.

The progressive motion of the moon's perigee, and the retrograde motion of its nodes, being both caused by the sun's attraction, are most rapid when the sun is in its perigee, and constantly grow slower and slower, till the sun reaches its apogee, where the motion becomes the slowest. Consequently, as the sun leaves its perigee, the moon's perigee immediately gets before, and its nodes behind their mean place, and continue so till the sun reaches its apogee, when, owing to the diminished rate of motion, their mean and true places again coincide. The contrary takes place when the sun is in the other half of its orbit. Hence it is apparent, that the moon's anomaly, being reckoned from its perigee, must be less than the mean when the sun's anomoly is less than 180°, and greater when greater; showing that something must be subtracted from the moon's anomaly in the former case, and added in the latter. The same must also be true of the longitude of the moon's nodes. These facts are indicated in Tables 6th and 7th, by the signs — and + placed at the head of the column containing the *argument*.

By the term *argument*, is meant that quantity on which others depend, and which determines their value. Thus, in this case, the sun's anomaly determines what correction must be applied to the moon's anomaly, or to the longitude of its node, and is, therefore, the *argument*.

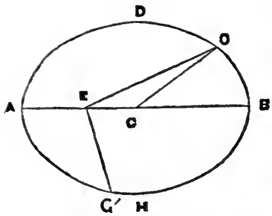
13. Entering tables 6th and 7th, with the sun's anomaly as an argument, we will take out the corrections which the foregoing considerations show to be necessary, and which are denominated Annual Equations of the moon's perigee and node, (taking care to make a proper allowance for the odd degree and decimals of the anomaly, as the tables give the equation only for every two degrees,) and apply the former to the moon's anomaly, and the latter to the longitude of the node, according to the sign + or — at the head of the column of the argument. Observe in these, and most of the other tables, that the unit figure of the argument is placed at the top or bottom of the table, and the other figures at the right or left. When the latter is found at the left, we must look for the former at the top, but when the latter is at the right, the former must be sought for at the bottom. Opposite the latter, and in the same column with the former, the equation is found. Thus, in our eclipse, the sun's anomaly being 143°.797, we look for the number 14 in the left hand column, and for 3 at the top. But, since the latter number is not found, the equation being given in the table only for 142° and 144°, we must note the difference between these equations, and take a proper proportion of it for the excess of the argument over 142°, viz. 1°.797. By this process, we find the annual equation of the perigee to be —0°.221; and of the node —0°.875. These, applied to the moon's anomaly, and the longitude of the node, make the former 145°.078, and the latter 61°.0008.

CHAPTER III.

EQUATION OF THE CENTRE.

14. The first inequality in the apparent motions of the sun and moon that claims attention, results from the eliptical form of their orbits, in consequence of which their motion is accelerated while passing from apogee to perigee, and retarded in the other half of their orbits, moving quickest in perigee, and slowest in apogee. The reason of this it is not difficult to discover.

Fig. 4.



Let AGH, &c. (Fig. 4) represent the moon's eliptical orbit, A and B the apsides, and E the earth. Let the moon start from perigee, at A, with its swiftest motion, and consequently with its greatest centrifugal force. The attraction of the earth, at E, not being able to retain it at that distance, it immediately begins to recede along the curve AGH. Being constantly pulled back by the earth's attraction, since the angle EGH is obtuse, its motion is retarded, and when it approaches the apogee, at B, its velocity has become so much diminished, that the attractive force of the earth prevents it from receding further. It thus arrives at apogee with its slowest motion. Leaving B with a weak centrifugal force, the superior attractive power of the earth at E, immediately begins to draw the moon toward itself along the curve BOD, constantly hurrying it onward at every point, as O, the angle EOD, contained between the direction toward which it is drawn, and that toward which it moves being now acute. By the time it reaches A, its velocity becomes so much increased, that it is prepared again to leave perigee in the same condition as at first, to pursue another similar round.

15. We shall arrive at the same conclusion, if we apply the principle, that when a body is retained in its orbit by a force directed toward a fixed point, as the moon is toward the earth in this case, the radius vector must describe equal areas in equal times. Hence the moon must move slower when it is near B, than when it is near A, in about the same ratio that its distance from the earth is greater. Not only is the absolute velocity greatest in perigee, but the angular velocity, with which only we are now concerned, is rendered still greater, by reason of the diminished distance.

The same reasoning will apply, in every respect, to the earth revolving in an eliptical orbit round the sun, and hence to the apparent motion of the sun round the earth.

16. If we consider the mean place of the sun and moon to be their true one, when in perigee, it will be, also, when in apogee; because each half of the orbit is described in the same time; but, as they start from perigee with their swiftest motion, they thus get ahead of their mean place, nor do they, though constantly retarded,

lose what they had gained, till the moment they arrive at apogee. Also, as they pass the apogee with their slowest motion, they directly get behind their mean place, and it is not till the moment they reach the perigee, that the continued acceleration of their motion, in this half of their orbits, enables them to gain up what they had thus lost. Consequently, they must always be ahead of their mean place, when in the former part of their orbits, and behind it in the latter. This difference between the mean and true place of the sun or moon, is termed the *equation of its centre.*

17. It is necessary to know in which half of their respective orbits the sun and moon are found at the time of our predicted eclipse: for, if either is moving from perigee to apogee, i. e., if its anomaly (4) is less than 180°, we must add something to its longitude already found, (11,) but subtract if in the other half of its orbit, i. e., if its anomaly is over 180°. The manner of computing the precise amount, will occupy our attention in another chapter. It is sufficient, here, to remark, that if, at any time, a line, OC, be drawn from the mean place of the moon to the centre of the elipse, the angle EOC, which this line makes with the radius vector, is very nearly equal to one-half the angular distance by which the moon is before or behind its mean place: so nearly that some authors have given this as a method of computing the equation of the centre. We shall use it hereafter as a convenient approximation. For the present, we will dispense with the labour of computation, and take the equation directly from tables already prepared.

By the calculations in the last chapter, (11 and 13,) we found that the sun's anomaly was 143°.797, and the moon's, as corrected, 145°.078. It appears, then, that each is moving from perigee to apogee, and is, therefore, ahead of its mean place, so that we must add something to their respective longitudes; with these anomalies, respectively, as arguments, we may now enter tables 8th and 9th, in the same manner as we did tables 6th and 7th, and take out the equation of the centre of the sun, and of the moon, applying the former to the the sun's longitude, and the latter to the moon's. The equations we find to be +1°.1165 and +3°.4154, and the resulting longitudes 65°.3494 and 67°.6483.

CHAPTER IV.

PERTURBATIONS IN THE MOON'S MOTION.—ANNUAL AND SECULAR EQUATIONS OF LONGITUDE.

18. In the preceding calculations, we first regarded the sun and moon as revolving in circular orbits, with uniform angular velocity; then, in elipses, describing equal areas by the radius vector, in equal times: but neither of these suppositions is strictly true. The sun's attraction disturbs the motion of the moon round the earth, producing numerous inequalities.

The method of calculating these, will be treated of in another place. It will be sufficient for our purpose, here, simply to give the theory of them, and then take the corresponding corrections from the tables.

19. The most obvious effect of the sun's attraction, is to draw the moon away from the earth, and thus enlarge its orbit. If this influence were always the same, it would occasion no inequality: but when the sun is in perigee, it is nearer to the earth, and consequently to the moon, than at other times; and the moon, therefore, will be more attracted by it. The moon being thus drawn farther away from the earth, when the sun is in this situation, and its periodic time consequently increased, it must fall behind its mean place. And although the attractive force of the sun diminishes as it leaves perigee, allowing the moon to contract its orbit and lessen its periodic time, it will not gain up what it had lost till the moment the sun reaches apogee. By similar reasoning, we may see that the moon must always be in advance of its mean place, so far as this cause is concerned, when the sun is in the other half of its orbit. There must then be applied to the moon's longitude a correction depending on the sun's anomaly; being additive when the anomaly is less than 180°, and subtractive when it is more. Such a correction is supplied in table 10th, entering which, with the degrees of the sun's mean anomaly, in the manner described in article 13th, we find the equation to be $-.1118$, which is to be applied both to the moon's longitude and anomaly,* for the cause we have been considering obviously affects both alike. The same is true of most

* The anomaly contains but three decimal places; hence, in applying the corrections to it, the 3d figure is given according to its nearest value.

of the other corrections that remain to be applied. The longitude, as already obtained, (17,) is 67°.6483, and the anomaly (13) 145°.078. Applying the above equation, they become 67°.5365, and 144°.966 respectively.

20. Connected with the foregoing inequality in the moon's motion, there is another of great historical interest, from the theories to which it formerly gave rise, viz., the acceleration of the moon's mean motion. It is too small to be discovered by direct observation, but becomes quite sensible in the lapse of ages.

21. Dr. Halley, wishing to know the precise length of a lunation, went back to the ancient Chaldean observations, intending to ascertain how many new moons had occurred between that time and his own, and then to divide the time by this number, which would give the average length of each. But he was surprised to find that a lunation in those days was considerably longer than now. By comparing the Chaldean, Alexandrian, Arabian, and the present observations, he found that the lunar period grew successively shorter.

22. Astronomers doubted the fact when it was first announced; but when they became satisfied of its truth, they set themselves to work to account for it. The most probable theory was, that the moon revolved in a resisting medium, which would cause it gradually to fall toward the earth, and thus, by reducing the size of the orbit, make the periodic time less. It must seem paradoxical to those who have not thought upon the subject, that such a cause could produce the effect in question; and that the retarding of the motion could make it revolve in less time. But it should be considered, that by diminishing the moon's velocity, its centrifugal force is diminished in a more rapid ratio, which would allow the earth to draw it nearer to itself, and reduce the size of the orbit. And it is demonstrable, that the gain in time from the latter circumstance, would more than counterbalance the loss from the former; so that on the whole, the moon's period would be shortened. The objection to this theory is, that comets, which are proved to be extremely light bodies, pass through this medium with little or no resistance. Hence it was inferred that the cause, if it exist-

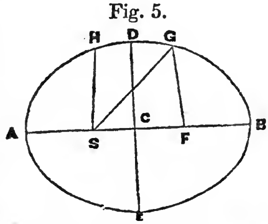
ed at all, was not sufficient to produce the effect of which we are speaking.

Other theories were advanced, but none were satisfactory; and it was reserved for La Place to explain the true reason of this acceleration of the moon's period, about sixty years ago.

23. Owing to the attraction of the other planets, the earth's orbit is gradually becoming less and less eliptical, or, nearer and nearer to a circle; so that the sun is every year about $39\frac{1}{2}$ miles nearer the centre of the elipse than it was on the year before. At this rate, the earth's orbit would become a circle in 40,315 years; an event, however, that can never take place, for long before such a period shall elapse, the change of which we are speaking, and which is only an inequality of long period, will have reached its limit, when the eccentricity of the orbit would again increase.

24. If it can be shown that the sun's attraction diminishes the moon's gravity toward the earth, and thus increases the periodic time, more than it would do if the earth revolved in a circle at the same mean distance, it is manifest that so long as the change in the shape of the earth's orbit, of which we have just spoken, goes on, the moon's periodic time must grow less and less.

Fig. 5.



Let ADBE (Fig. 5) represent an eliptical orbit, S the attracting body, placed in one of the foci, C the centre of the elipse, and F the other focus. It can be demonstrated, that the mean distance of S from all points in the orbit, is equal to AC or CB. Take any two points in the orbit G and H, equidistant from B and A. We propose to prove that the average attraction of S upon the moving body, when at these points, is greater than it is when the body is at its mean distance. And since these are any points in the arcs AD and DB, if we prove it for them, we prove it for the whole orbit.

Join GS, GF and HS, and let SG bear any ratio, other than that of equality, to GF; say 6 : 4. Then, since by the properties of the elipse SG+GF=AB=2CB, it follows that the ratio of SG to CB is 6 : 5, and of GF, or its equal HS, to CB, 4 : 5. Therefore, since the force of gravity is inversely proportioned to the square of the distance, the attraction at the former point will be $\frac{25}{36}$, and at the

latter $\frac{25}{16}$ of what it is at the mean distance. The average between them is $\frac{325}{288}$ of the attraction at the mean distance,—exceeding it by $\frac{37}{288}$.

Now the earth's orbit is much nearer circular than we have supposed this to be, and the excess of attraction must be proportionably less: but still there must be an excess, so long as it is eliptical at all. Hence, as the earth's orbit becomes nearer circular, the sun's attraction upon it, and consequently upon the moon, must continually grow less, allowing the orbit of the latter to contract. This would diminish the periodic time, and produce the very effect that excited so much wonder in the mind of Dr. Halley, and the astronomers of his time.

25. The tables for this work are based on the moon's motion, as it existed in the year 1800, and we must, therefore, add to its longitude and anomaly the amount gained since that time, from the cause just explained. Table 11th contains the required correction, calculated at intervals of five years, during the present century. Look for the year in the left hand column of the table, except the unit figure, which is placed at the top, and opposite to the former, and under the latter will be found the correction required, expressed in decimals of a degree, the first two places, which are ciphers, being omitted.

The correction for the year 1854 is .0009, which, added to the longitude and anomaly already found, (19,) makes the former 67°.5374, and the latter 144°.967.

CHAPTER V.

PERTURBATIONS IN THE MOON'S MOTION, CONTINUED.—VARIATION.

26. THE moon's motions grow more complicated the farther we proceed. To investigate them thoroughly, is nothing less than a solution of the famed Problem of the Three Bodies. The moon's orbit, which we first regarded as a circle, and then an elipse, we shall now find to be neither a circle nor an elipse,* but an irregular

* This statement seems to conflict with former ones, where the eliptical form of the moon's orbit was asserted; but its mean shape was then intended, without taking into account the irregularities.

oval shaped figure, which is constantly changing its form. The prospect before us, in trying to reduce such irregularities to order, so as to see their precise influence on the moon's longitude, is sufficiently appalling, but nevertheless, let us not be deterred from the attempt.

To avoid misapprehension, it ought perhaps here to be remarked, that these irregularities are not of such a nature as to set aside our previous work, but only show that, under some circumstances, they may occasion necessary corrections.

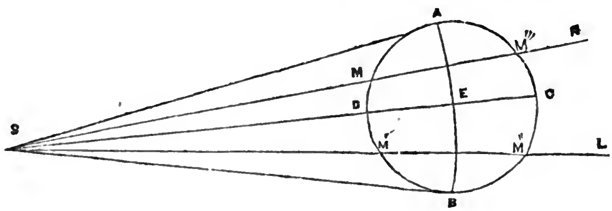
27. And first let us, in this chapter, see what the shape of the orbit would be, and how the moon would revolve in it, on the supposition that it was originally a circle round the earth, but drawn out of shape by the sun's attraction. We shall in this way discover the cause of an observed inequality in the moon's motion, denominated variation, and discovered by Tycho Brahe, A. D. 1590. The modifications that the orbit would undergo, by supposing the original figure an elipse instead of a circle, will occupy our attention in the next chapter.

28. Let S (Fig. 6) represent the sun, E the earth, and ADCB the moon's orbit; and let us suppose, for a moment, that the moon retains a circular orbit. Let D represent the place of the moon at conjunction, C at apposition, and A and B when it is at the same distance from the sun that the earth is, or very nearly in quadrature.

First, let the moon be at A or B, in which case the moon and earth being equally distant from the sun, must be equally attracted by it, and consequently there would be no tendency to change their direction from each other, but only to draw them nearer together, which would be precisely equivalent to *increasing* the earth's power of gravity.

Next let it be in conjunction at D. Now the earth and moon are both in the same direction from the sun; but the moon being nearest is more attracted, in the inverse ratio of the square of the distance, i. e., $SE^2 : SD^2$. The only effect, therefore, is to draw the moon directly away from the earth, by virtue of the difference in the attractive forces, which would be equivalent to *diminishing* the earth's attraction.

Fig. 6.



29. Again,—suppose the moon at any point M in the quadrant AD. Being nearer to the sun than the earth is, it is more attracted by it, and the effect is nearly* the same as though the earth was not attracted at all, but the moon drawn along the line MS by a force equal to the difference of the attractions. The direction of this force, making an acute angle with that in which the moon moves, must accellerate the motion in its orbit; and the same would be true of every point in the quadrant AD. If the moon were at M′ any point in the quadrant DB, the difference of attractions acting along the line M′S, would tend to retard its motion.

Once more: let the moon be at any point M″ in the quadrant BC. Being further from the sun than the earth is, it is less attracted by it, which is nearly as though it were drawn in the opposite direction, along the line M″L. The effect would be to accelerate the motion, in nearly the same manner as in the quadrant AD. If the moon was at any point M‴ in the quadrant CA, the difference of attractions acting, as it were, along the line M‴N, would retard its motion.

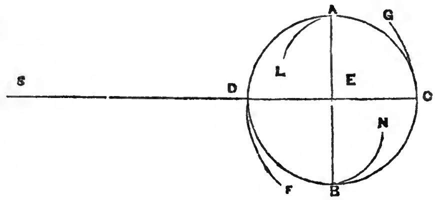
Thus the moon is alternately accelerated and retarded in the differant quadrants; moving swiftest in syzygy and slowest in quadrature. Hence, from this cause alone, the moon would be in advance of its mean place while passing from syzygy to quadrature, and behind it while passing from quadrature to syzygy.

30. But the above is not the only reason. We have thus far, in the present investigation, supposed the moon's orbit to retain its circular form, notwithstanding the disturbing influence of the sun: but this is not possible. To retain a body in a circular orbit, the centripetal and centrifugal forces must be equal. But we have just seen that the velocity in syzygy, as at C and D, (Fig. 7,) is

* Sufficiently near for our present purpose. The subject will be investigated more critically hereafter.

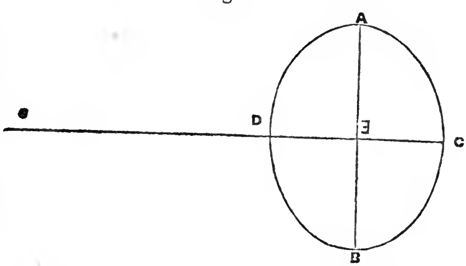
greater than at A and B: and as the centrifugal force is proportioned to the square of the velocity, it must be greater. On the other hand, it was shown (28) that the sun's attraction diminished the moon's gravity toward the earth in syzygy, as at C and D, and increased it in quadrature, as at A and B. Taking both these facts into consideration, it is manifest that at A and B, the centripetal force must considerably exceed the centrifugal, while at C and D, the centrifugal will be the greatest, which would cause the moon's track to fall within the circle at the former points, as to L and N, and without it in the latter, as to F and G.

Fig. 7.



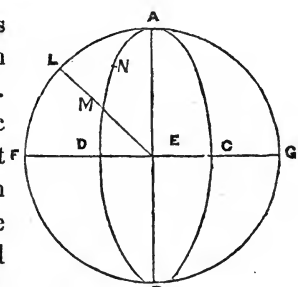
The effect would be to throw the orbit into something such a shape as is represented in Fig. 8, viz: a kind of oval, with its longest diameter, AB, at right angles to line ES, drawn from the earth to the sun.

Fig. 8.



Will this alteration in the the shape of the moon's orbit affect its longitude? To aid us in this investigation, we will circumscribe the oval by a circle; and to make the illustration more striking, we will suppose the oval very much flattened, so as nearly to coincide with AB, as in Fig. 9. Now, if the arc AF be divided in any given ratio at the point L, and LE be drawn, it will cut the arc AD by no means in the same ratio. AM will bear a much greater ratio to MD than AL to LF. Hence, if two bodies, whose periodic times were equal, should start from A at the same time, and move with uniform velocity, one in the circle ALF, and the other in the oval AMD, the former would arrive at L before the latter would at M, leaving it behind perhaps at N. The same reasoning may be applied to the other quadrants, though with the opposite effect in DB

Fig. 9.



and CA, when it will show that the moon must be in advance of its mean place.

31. There are two reasons, then, why the moon will be behind its mean place when passing from quadrature to syzygy, but in advance of it while passing from syzygy to quadrature; 1st, from its unequal motion, (29,) and 2d, from the shape of its orbit. The maximum effect of the former to change the moon's place, is from 9′ 17″ to 10′ 15″, and the latter from 23′ 56″ to 26′ 52″, according to the distance of the earth from the sun. When at its mean distance, the maximum effects are 9′ 46″ for the former, and 25′ 24″ for the latter, amounting to 35′ 10″ for both united.

32. If the four quadrants were perfectly symmetrical, a table showing the correction required for each degree in one quadrant, would answer for all the rest; only the equation would be additive when the moon is passing from syzygy to quadrature, i. e., in the arcs DB or CA, and subtractive when it is passing from quadrature to syzygy, i. e., in the arcs AD and BC. But there is a slight difference; for, 1st, the disturbing influence is a little less in the half of the orbit nearest the sun than in the other half, the difference of the squares of the distance of the earth and moon from the sun, being a trifle less; and 2d, the quadrants (so termed for the sake of conciseness) nearest the sun contain a little less than 90° each, and the other two quadrants, each a little more than 90°, for AB is not strictly a straight line, but an arc of the earth's orbit. A table for two quadrants would, however, be sufficient—one in the half of the orbit next to the sun, and the other in the half most remote from it, as, for example, DB and BC. Table 12th is constructed in this way, where it will be seen that the equations are additive for a little less than 90° after the moon leaves D, and then subtractive to the end of the next quadrant. If the moon's angular distance from the sun exceeds 180°, which would carry it into the quadrants CA or AD, the degrees are found at the right hand and bottom of the table, and direction is given to "reverse the signs," so that the equations which were additive in DB become subtractive in AD, and those which were subtractive in BC become additive in CA.

33. The angular distance of the moon from the sun, (found by subtracting the longitude of the latter from that of the former, as thus far corrected, borrowing 360° if necessary,) shows in which quadrant the moon is. When the difference is from 0° to 90°, or from 180° to 270°, the moon is passing from syzygy to quadrature, but when it is from 90° to 180°, or from 270° to 360°, the moon is passing from quadrature to syzygy. In the present case, the longitude of the sun (17) is 64°.3494, that of the moon (25) 67°.5374, and the excess of the latter 2°.1880. Entering table 12th with this argument, in the same manner as directed in article 13th, the equation is found to be +.0445. This is to be applied to the moon's longitude and anomaly according to its sign. If the argument had been over 180°, the sign of the equation would have to be changed to —. After this equation is applied, the moon's longitude becomes 67°.5819, and the anomaly 145°.012.

34. The inequality to which this chapter is devoted, being occasioned by the disturbing influence of the sun, must be more or less according as the distance of that luminary varies, as we have already observed, (31.) In table 12th, and, consequently, in the equation that was just applied, the sun is supposed to be at its mean distance. Hence another correction becomes necessary, which must evidently depend on the same circumstances as the last, together with another, viz., the distance of the sun from the earth, which is determined by its anomaly. Accordingly in table 13th two arguments are employed; viz., 1st, the argument just used for variation, which is to be sought for at the top or bottom of the table, and 2d, the sun's anomaly at the right or left. If the former is found at the top, we look for the latter at the left; but if at the bottom, at the right. The equation is found opposite the latter, and in the same column with the former. Since one argument is given only for every 5° and the other for 10°, it is necessary to institute a kind of double proportion for the units and decimals. It is further to be noticed, that if both arguments are to be found in the same gnomon, enclosed by the heavy lines about the table, the equation is to be applied with its proper sign, as found in the table; but if one is found in the inner and the other in the outer gnomon, the sign before the equation is to be changed from + to —, or

from — to +. In the present case, the former argument (33) is 2°.1880, which being between 0° and 5°, is to be considered as found in the inner gnomon at the top; and the latter (11) is 143°.797, which is found in the outer gnomon, at the left. Making a proper allowance for the units and decimals, the equation is +.0053; but the arguments being found, one in the *inner* and the other in the *outer* gnomon, the sign must be changed, and the equation becomes —.0053. This applied to the moon's longitude and anomaly, found in the last article, makes the former 67°.5766, and the latter 145°.007.

CHAPTER VI.

PERTURBATIONS IN THE MOON'S MOTION, CONTINUED.—EVECTION.

35. The inequality which is to occupy our attention in this chapter was discovered by Ptolemy, A. D. 110, and is denominated Evection.

For distinctness of conception, it is necessary to bear in mind the precise difference between this correction and that treated of in the last chapter, for there is danger of confounding them, since both are caused by the disturbing force of the sun in the plane of the ecliptic. That supposed the original form of the moon's orbit a *circle*, this an *elipse*, and wholly dependent on its eccentricity; so that if the elipse had no eccentricity, there would be no correction for evection. That always elongated the orbit in the direction of the *quadratures;* this, we shall see, elongates it in the direction of the *syzygyes.* That regarded the shape of the orbit as *constant;* this, as ever *changing.* An important element in this correction is the irregular motion of the line of apsides; that had no such line to take into account.

36. It will be shown, that the progressive line of the moon's apsides is quite irregular; that it sometimes progresses more and some-

times less rapidly; sometimes remains stationary, and sometimes even goes backward. Now in determining the moon's mean anomaly, (11,) all the motions were supposed uniform, and no correction has been made for any irregularity in the motion of the moon's perigee, except that which resulted from the unequal distance of the sun, (13.) But since the anomaly is reckoned from the perigee, it must be subject to all the irregularities that the perigee itself is. Hence, in applying the equation of the centre, (17,) we used data that were erroneous, and the error that was introduced needs to be corrected.

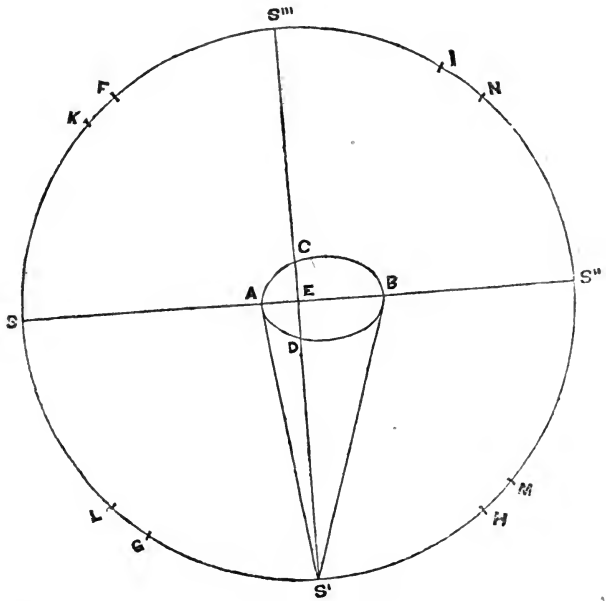
27. But this is not all. The greater the eccentricity of an orbit is, the greater is the equation of the centre. Thus the equation of the moon's centre is much greater than that of the sun with the same anomaly, (compare tables 8th and 9th,) because the orbit of the former is much the most eccentric. Now it will be shown presently, that the eccentricity of the moon's orbit is ever varying, and the equation of the centre, which depends upon it, must vary likewise; whereas table 9th is computed on the supposition of a constant mean eccentricity. So that we not only made use of a wrong anomaly in applying the equation of the centre, but also a wrong eccentricity in the moon's orbit. The correction for the combined effect of these two errors constitutes *evection.*

38. It will be demonstrated in its proper place, that if a revolving body be retained in an eliptical orbit, by a force directed toward one of the foci, the square of the distance of the body from that focus, at any point in its orbit, must always be inversely proportioned to the intensity of the attractive force at that point. Hence, if an in-increase or diminution of attraction were to take place throughout the orbit, proportional to the existing attractions at each point, the size, but not the form of the orbit would be changed. The eccentricity would remain the same as before. But if the alteration in the attractive force were in any other ratio, it obviously would affect the shape of the orbit. If the perigeal gravity was made to bear too great a ratio to the apogeal, it would be drawn in too much at the former point, or too little at the latter, and the orbit would become more eccentric. Or if the apogeal gravity became

too great in proportion to the perigeal, the orbit would be rendered less eccentric, or more nearly circular.

39. To apply this to our subject, let E (Fig. 10) represent the earth, ADBC the moon's orbit, A being the perigee and B the apogee, and FGHI the sun's apparent orbit. First, let the sun be at S, so that the line of apsides, AB, of the moon's orbit is directed to-

Fig. 10.



wards it, or lies in syzygy. The moon is more attracted by the earth at A than it is at B, in the inverse ratio of AE^2 to EB^2; and in order that the disturbing influence of the sun, which tends (28) to diminish the earth's attraction at these points, should effect no change in the shape of the moon's orbit, it must also (38) be more at A than at B, in the same ratio. But instead of that, the sun's disturbing influence is greater at B than at A, for the difference between SE^2 and SB^2 is greater than between SA^2 and SE^2; consequently the relative difference in the attractive forces at A and B toward E is increased, and the orbit must become more eccen-

tric. In the same manner it may be shown that the eccentricity of the moon's orbit must be increased when the sun is at S″.

But if the sun were at S′, and the moon at A or B, the latter would be drawn toward the earth by the sun's disturbing influence, and its gravity increased, (28 ;) but more at B than A in the ratio of EB to EA, as will appear if we resolve the force in the direction S′A into two others in the directions AE and ES′, and that in the direction S′B into two in the directions BE and ES′. In this case the greatest addition is made to the least force, whereas to preserve the shape of the orbit unchanged, the additional gravities should be in proportion to the previously existing ones. The apogeal gravity thus becomes too great in proportion to the perigeal, and the eccentricity of the orbit is diminished, (38.) We shall arrive at the same conclusion if the sun be supposed to be at S‴.

Thus the eccentricity of the moon's orbit is greatest when the line of its ipsides lies in syzygy, and least when it lies in quadrature. It is plain that these changes in eccentricity occur, not instantaneously, but gradually, as the sun progresses in its orbit. The eccentricity must diminish while the sun is passing from S to S′, or from S″ to S‴, and increase while it is passing from S′ to S″, or from S‴ to S, being at its mean state when the sun is about half way between these points, as at L, M, N and K. The eccentricity must exceed the mean when it is in the quadrants KL, or MN, and be less than the mean when it is in the quadrants LM and NK.

40. The investigation of the irregular motion of the line of the apsides of the moon's orbit, on which the evection in part depends, is considerably more difficult than any of the preceding, and will require the reader's close attention. That it must, on the whole, progress, will appear, when we consider that the average effect of the sun's attraction is to draw the moon away from the earth, and thus to render its orbit less curved than it would otherwise be. Consequently, after the moon leaves its perigee, or apogee, where its motion is at right angles with the radius vector, its angular motion round the earth must amount to more than 180° before its path will have been deflected enough to intersect the radius vector at right angles again. That is, it must be more than 180° from

perigee to apogee, or from apogee to perigee.* And, further; since the attraction of the sun sometimes increases, and sometimes diminishes the moon's gravity toward the earth, we should conclude the line of its apsides must sometimes regress and sometimes progress. We must, however, go into a more minute investigation of this motion, to account for all the phenomena to which it gives rise.†

41. *If the moon, or any other body revolving in an eliptical orbit, should be deflected from its natural course at any point by some disturbing influence, so as to move at right angles to the radius vector, the point where such deflection occurred would thenceforward become one of the apsides of the orbit, provided it were not further disturbed.*

This is evident, from the fact that the apsides are the only points in an eliptical orbit, where the curve is at right angles to the radius vector. Also the body must still revolve in an elipse, or some other conic section, for we shall demonstrate hereafter that simple gravitation toward a fixed point can retain it in no other curve. Whether the point of deflection will be the perigee or the apogee, will depend on the velocity of the time; if it be greater than the mean, the point will be the perigee, but if less, the apogee.

42. To apply this principle, let us inquire what alteration must be made in the attractive force of the earth, E, (Fig. 11.) to bring the motion of the moon at right angles to the radius vector at the points of its orbit C, D, F, and G, the two former being near the perigee, A, where the velocity exceeds the mean, and the two latter near apogee, B, where it is less. While the moon is moving from A, through S to B, the direction of its motion constantly makes an obtuse angle with the radius vector, (as EFM;) it would be

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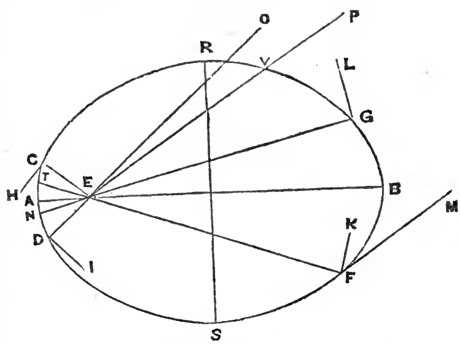
* Playfair.

† The articles between this and the 56th may be omitted, if the instructer should deem it expedient.

Fig. 11.



necessary, therefore, at the point F, that the attractive force of E should be *increased,* to curve its motion to K, and thus bring it at right angles with EF. And, if it were so increased, the point F would (44) henceforth become the apogee, instead of B. In other words, the apogee would have moved *backward* from B to F; and consequently the perigee from A to T, for they must always be opposite each other.

The same reasoning will apply to the point D; yet, if the deflecting force should occur there, D would become the perigee instead of the apogee, on account of the moon's greater velocity at that point, (44,) and the apogee would be found in the direction of the line DO; so that the apsides would have moved *forward* from A to D, and from B to O. In the other half of the moon's orbit, where the direction of the motion continually makes an acute angle (as EGR) with the radius vector EG or EC, it is manifest, that the attraction of E must be *diminished,* in order to bring the motion at right angles, as GL and CH; or, rather, a repulsive force must be given to it. Such a deflection occurring at G, would change the place of the *apogee* from B to G, or, would make it move *forward.* If occurring at C, it would change the place of *perigee* from A to C, or, would make it move *backward.*

43. If instead of increasing the gravity at F, it were diminished, it is pretty clear that the apogee would move *forward* instead of backward. To illustrate it, let us suppose the gravity to be greatly diminished, so much so as to be nearly destroyed. The moon, being scarcely attracted at all toward E, will fly off nearly in a tangent to the elipse at F, and the apogee will be found in that direction, but infinitely distant. If then, we draw NP parallel to the tangent FM, it will point to the place of the apogee, which has, therefore, moved *forward,* equivalent to the arc from B to V. If the attraction were less diminished, it would not move forward so far, but the reasoning would hold good.

Similar reasoning applied to the points D, C, and G, will show that, by reversing the supposed alteration in the force of gravity at those points, we shall reverse also the motion of the line of the apsides.

Summing up our conclusions, we find that an *increase* of the moon's natural gravity toward the earth, near apogee, on either side, would cause the line of apsides to *regress;* while a *diminution* would cause it to *progress;* and that the reverse takes place by an alteration in the natural force of gravity when the moon is near its perigee. I employ the terms *natural gravity,* and *natural velocity,* to signify the gravity and velocity that the moon would have if it revolved regularly in its eliptic orbit, undisturbed by the attraction of any foreign body.

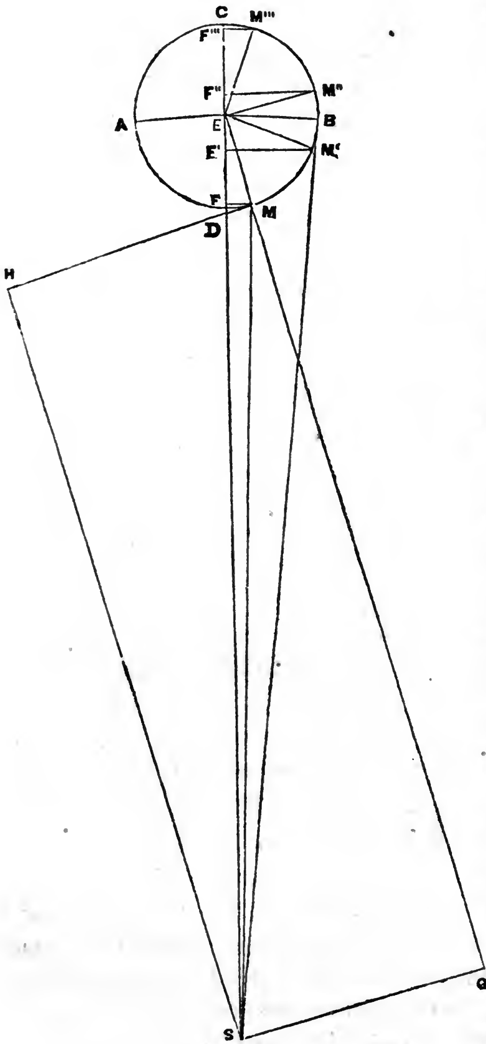
44. It is evident that an increase in the moon's velocity, and consequently of its centrifugal force, must have nearly the same effect as a diminution of the earth's attraction; and *vice versa.* Hence, if its velocity near apogee is, from any cause, rendered greater than its natural velocity in that part of its orbit, the line of apsides must move forward, or progress; and the reverse, if such an increase of velocity occurs near perigee. If both these causes conspire, (and we will proceed to show that they do,) the progress or regress of the apsides must be still more rapid.

45. It has been shown, (28,) that the sun's disturbing influence increased the moon's gravity toward the earth in quadrature, but diminished it in syzygy; and we should suppose that there must be intervening points, where it exerted no influence either way. These points it is important for us to find, for when the moon is at these, the apsides of its orbit must be at rest, so far as their motion is caused by a variation in gravity.

46. Let the moon be at any point M (Fig. 12) of its orbit, and let the sun's attraction on it at that point be represented by *m.* Resolving this force into two others in the directions ME and ES, the proportion for the former (called the *addititious* force, because it increases the moon's gravity toward the earth) will read SM : ME : : *m* : *the addititious force* $=\frac{\mathrm{ME}}{\mathrm{SM}}m=\frac{\mathrm{ME}^2}{\mathrm{SM+ME}}m$.

47. The proportion for the latter force, in the direction ES, will read SM : ES : : m : the force required $=\frac{ES}{SM}m$. But the earth is attracted by the sun in the same direction, ES, and it is the difference of the attractions only that exerts any disturbing influence on the moon in this direction. We will therefore find how much the attraction of the sun on the earth is, and subtract it from that just found, viz., $\frac{ES}{SM}m$. By the laws of gravity $ES^2 : SM^2 :: m$: the sun's attraction at the distance ES. Hence the earth is attracted with a force equal to $\frac{SM^2}{ES^2}m$, which is to be subtracted from $\frac{ES}{SM}m$. Reducing the fractions to a common denominator, and subtracting, we have $\frac{ES^3-SM^3}{ES^2+SM}m$. Now SM$=$ ES—EF, very nearly; therefore, by involving both sides, and rejecting the 3d and 4th terms in the right hand member on account of their smallness, we have $SM^3 = ES^3 - 3ES_2 \times EF$. Substituting this value in the place of SM^3, the above fraction, which expresses the disturbing influence of the sun in the direction ES, becomes $\frac{3ES^2\times EF}{ES^2\times SM}m = \frac{3EF}{SM}m$. Resolving this force into two others, one in the direction EM, and the other at right angles to it; i. e., in the directions EG and SG, the proportion for the former (called the *ablatitious* force, because it diminishes the moon's gravity toward

Fig. 12.



the earth) will read SE : EG, or (since the triangles EGS and EFM are similar) ME : EF : : $\frac{3EF}{SM}m$: the *ablatitious force*$=\frac{3FM^2}{SM\times ME}m$.

48. The addititious and ablatitious forces acting in direct opposition, must neutralize each other at the points in the orbit where they are equal, showing that, at such points, the sun's attraction produces no effect on the gravity of the moon toward the earth. In that part of the orbit that lies between these points and quadrature, there will be an increase of gravity, and between those points and syzygy, at C or D, (for the demonstration will apply to either half of the orbit ACB or ADB,) a diminution.

49. But since the fractions representing these forces have a common denominator, they will evidently be equal when their numerators are equal, i. e., when $3EF^2=ME^2$; or (extracting the square root) when $\sqrt{3}\times EF=ME$; or (converting the equation into a proportion) when ME : EF : : $\sqrt{3}$: 1. But ME : EF : : 1 : cos. MES; therefore, by equality of ratios, $\sqrt{3}$: 1 : : 1 : cos. MES$=$ .5773672, which is the cosine of 54° 43′ 56″. Hence the gravity of the moon toward the earth is diminished when it is within 54° 43′ 56″ of syzygy, and increased when it is within 35° 16′ 4″ of quadrature.

50. It is worthy of notice here, that the diminution of gravity in syzygy is about double the increase in quadrature. The above reasoning shows, that the ablatitious force is to the addititious as $3EF^2 : ME^2$. But in syzygy EF=ME, and $3EF^2=3ME^2$; so that the difference between them is $2ME^2$; while, in quadrature, EF becomes 0, and the ablatitious force disappears, leaving the addititious force proportional to ME^2, which is half of $2ME^2$.

51. There remains, not yet investigated, the tangential force in the direction GS, or MH, one of the parts into which we resolved (47) that in the direction ES. Its precise amount it is not now material for us to know; but it is to be observed, that its only influence is to retard the moon's motion from D to B; since, being at right angles to EM, it neither increases nor diminishes the moon's gravity toward the earth. If the moon was supposed to be in any of the other quadrants, and figures constructed on the same principle as this, we should see that the moon must be retarded in pass-

ing from D to B, and from C to A; but accelerated from A to D, and from B to C. Hence its motion must be swiftest in syzygy at C and D, slowest in quadrature at A and B, and a mean half way between syzygy and quadrature. This is the same conclusion to which we arrived by a less rigid process, in article 29th.

52. To show how the motion of the line of apsides is affected by this perturbation in the moon's gravity and velocity, let us recur again to Fig. 10th. Let the sun be at S, or S″, so that the line of apsides, AB, lies in syzygy. In this case, both the moon's velocity in its orbit will be increased, and its gravity towards the earth diminished (48) at A and B. Consequently, (43 and 44,) the line of apsides must move forward when the moon is near apogee, at B; but backward, when it is near perigee, at A; and if the regress near perigee is equal to the progress near apogee, they will balance each other, so as, on the whole, to produce no change in the position of the line of apsides. But they are not equal, for several reasons.

1st. The diminution of the moon's gravity at these points, and the increase of velocity, are both caused by the force in the direction ES, (Fig. 12,) which was obtained in article 47, by taking the the difference of the attractions of the sun upon the earth and moon, in that direction. Consequently, they must depend on the difference between the distances of the moon and earth from the sun; and this difference is greater when the moon is in apogee, than when it is in perigee.

2d. The forces causing the line of apsides to progress, act for a longer time than those causing it to regress, because the moon is longer in describing the apogeal than the perigeal half of its orbit.

3d. A given force would produce more effect on the moon when it is in apogee than when it is in perigee, on account of its *natural* velocity being less at the former point, and therefore more easily deflected from its orbit. If a cannon ball were moving but one foot in a second, it would not be very difficult to turn it out of its course, but not so if it were moving 1000 feet per second.

From all these circumstances combined, the line of apsides progresses quite rapidly when the sun is at S, or S″. And, since the same causes operate in the same way, one (51) while the sun is passing through the arcs KL and MN, extending 45° each way from S and S″, the direction of the line of apsides of the moon's

orbit, and the other (49) through FG and HI, extending 54° 43′ 56″ from the same points, it follows that the line of apsides must progress, at least, while the sun is in the quadrants KL and MN.

53. When the sun is in either of the small arcs FK, LG, HM or NI, containing 9° 43′ 56″ each, the line of apsides is nearly stationary: for the perturbations, both in gravity and velocity, being near their limits are very weak, and what small force they do exert is in opposition to each other, the former tending to make the apsides progress, and the latter, regress.

54. Now let us suppose the sun at S′ or S‴, so that the line of apsides shall be at right angles to ES′, or shall be in quadrature. The moon's gravity toward the earth, when at A and B, will now be increased, (48,) and its velocity diminished, (51.) Consequently, (43 and 44,) the line of apsides must progress when the moon is near perigee, but regress when it is near apogee. And, by nearly the same reasoning as employed above, (52,) it may be shown, that the regress exceeds the progress; so that, on the whole, the line of apsides regresses. In like manner, it may be shown, that it regresses, though less rapidly, when the sun is any where in the arcs IF or GH.

55. The regress here will, however, be less rapid than the progress when the sun is in the arcs KL and MN, for the perturbation in the moon's gravity is (50) but half as great. It will also be of shorter duration, for the arcs IF and GH contain but 70° 32′ 8″ each, and the sun moving forward in its orbit about 1° per day, while the line of apsides moves backward, on an average, about 1·6 of a degree per day, each arc will be passed over in about 61 days; when the line will become nearly stationary for about 10 days, (the time occupied in passing one of the small arcs, as FK,) and then begin to advance.*

* It may seem to the reader erronious, to ascribe any part of the progress of the moon's apsides to the perturbation in velocity; for, since that is equal in quadrature and syzygy, and extends the same distance, 45°, from each, it would seem that, so far as this cause is concerned, the regress, when the apsides are near quadrature, should be just equal to the progress when they are near syzygy. Sir Isaac Newton took this view of the subject, and was greatly perplexed at finding that he could account for but about half the motion of the line of the apsides. The explanation here given, is, in substance, that of Clairaut, who showed that, when the apsides regressed, they approached to meet the sun, thus shortening the regressive arc, and, consequently, diminishing the perturbation in velocity; but, when they progressed, they receded from the sun, lengthening the progressive arc, and thereby increasing the perturbation in velocity. So that the perturbation would not only be greater in the latter case than in the former, but extend through a greater arc.

Since the line of apsides remains nearly stationary about 40 days in the year, moves backward about 122, and forward during the remainder, amounting to about 203 days; and since its forward motion is more rapid than its backward, it is evident that, in the course of a year, it must, on the whole, advance. The rate of advance is found by observation, to be such as to carry it entirely round the orbit in 3232 days, 13 hours, 48 minutes, and 29.4 seconds, or about nine years.

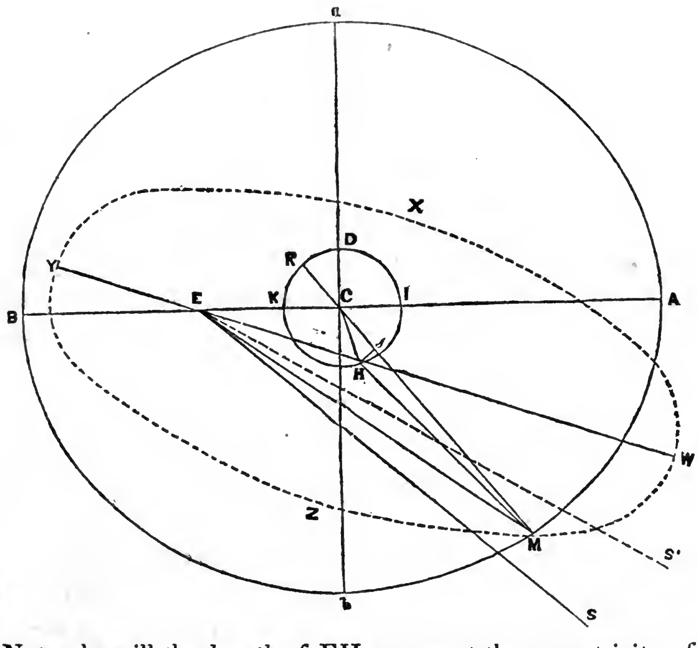
The foregoing explanations have, I trust, made the theory of the variation in the eccentricity of the moon's orbit, and the irregular motion of the line of its apsides tolerably clear to the reader. It remains to make a practical application of it.

56. We have seen (39, 52 and 54) that the eccentricity of the moon's orbit exceeds the mean, when the sun is in those quadrants where it causes the line of the moon's apsides to advance, and is less than the mean when the sun is in the other parts of its orbit: also, that these changes occurred alternately, and nearly in alternate quadrants, the lines of division being not far from half way between syzygy and quadrature. We will eneavour to represent these changes by a figure, the construction of which was devised by Sir Isaac Newton for the purpose, and which observation shows to be very nearly correct.

Let AaBb (Fig. 13) represent the moon's eliptical orbit, in its mean state, M the moon, E the earth, placed in one of the foci, and EC the mean eccentricity. Now, as the eccentricity varies, the centre C will sometimes approach toward E, as far as K, and sometimes recede from it to I; the distances CK and CI being the greatest variation of the eccentricity from the mean. Describe the circle IDKH, and join MC and ME. Let ES represent the direction of the sun, and B the moon's mean perigee, i.e., the place of the perigee if it progressed uniformly. Now, since it has been shown (39) that when ES is at right angles to AB, the eccentricity is least, and when it coincides with it, the greatest, it is plain that it must be represented by EK in the former case, and EI in the latter. And when ES is in any other position, the eccentricity must be represented by a line longer than EK, and shorter than EI. We shall effect this for every possible position of ES, by always making the angular distance of H from I, in the direction IDKH, equal to twice the angle BES. The length of EH will be

equal to the eccentricity at the time, very nearly. For example, when BES=45°, H will coincide with D, making EH very little longer than EC, and thus showing that the eccentricity exceeds the mean in the same small ratio. Again, if BES=90°, H would coincide with K, showing that the eccentricity was now a minimum, which agrees with what we have already seen to be true. In the same manner it may be shown, that at any other point, this construction brings out very nearly the result it should do according to our previous reasoning.

Fig. 13.



Not only will the *length* of EH represent the eccentricity of the moon's orbit at all times, but its *position* will represent that of the line of apsides very nearly, never varying from it more than 3′. Hence, the point H may always be considered as the centre of the eclipse; thus changing the whole eclipse from the mean position AaBb, to that represented by the dotted curve WXYZ.

57. It was remarked, (17.) that if two lines were drawn from the mean place of the moon, one to the centre of the elipse, and the other to the focus, round which it revolved, the angle at the moon,

contained by these lines, would be equal to half the equation of the centre, very nearly. If the eccentricity and position of the elipse remained unchanged, the angle in question would be EMC; but when, in consequence of the change, H becomes the centre of the elipse, the angle becomes EMH. Therefore HMC, which is the difference between these two angles, must be half the effect of the disturbing force of the sun; or, in other words, it is half the evection we have been so long in quest of. Hence, if we can find out a method of determining the size of this angle, or the conditions on which its size depends, our task is over, for by doubling it we shall have the correction required.

58. Draw H*s* at right angles to MC. Then, since small angles are nearly proportional to their sines, the line H*s* must always be nearly proportional to the angle HMS, and consequently to the evection. But H*s* is also the sine of HC*s*, or its supplement HCR; therefore the evection is always proportional to the sine of HCR. This angle we will proceed to find. The angle SEB=MEB—MES; therefore the angular distance of H from I, in the direction IDK, (being, by construction, double of SEB,) =2MEB—2MES. The angles MEB and MCB are nearly equal,* the eccentricity of the orbit being small; therefore, subtracting MCB, or its equal ICR, from the first member of our equation, and MEB from the last, we have HCR=MEB—2MES. Now MEB is the moon's mean anomaly, and MES is the angular distance between the sun and moon, or the excess of the mean longitude of the moon over the true longitude of the sun. Hence the evection, which has been shown to be proportional to the sine of HCR, is proportional to the sine of the moon's mean anomaly diminished by twice the excess of its mean longitude over the true longitude of the sun.†

* There is danger that the proportions of the different lines, as they appear in the figure, may mislead the reader, and it is well to remember, that EC is but about 1-20, and KC about 1-100 of CB.

† In this demonstration, the moon's longitude is supposed to exceed that of the sun. But we shall arrive at the same conclusion if we suppose the sun's longitude the greatest; as, for example, if it be in the direction ES′. For, now, S′EB=MEB+MES′; and, consequently, by the same construction and reasoning as in the other case, HCR=MEB+2MES′. But the result is obviously the same, whether we add the angle MES′, or subtract its supplement; that is, the angular distance of M from S′, reckoned in the other direction, S′A*a*BM. Now, this supplementary distance is the excess of the moon's longitude over that of the sun, borrowing 360°, or one revolution; therefore, the angle HCR is still equal to the moon's mean anomaly, diminished by twice the excess of its mean longitude over the true longitude of the sun; and the principle becomes general in its application.

If the reader will now turn to table 14th, he will notice that this is the argument by which the evection is taken out in that table.

59. At the time of our predicted eclipse, the quantities are as follows :—

The moon's mean anomaly is, (13,) - - -		145°.0780*
The moon's mean longitude is, (11,)	64°.2329	
Subtract sun's corrected longitude, (17,)	65 .3494	
	358 .8835×2=	357 .7670
Argument of evection, - - - - - -		147 .3110

60. Entering table 14th with this number as an argument, in the same manner as heretofore, the required correction is found to be .7156, which the sign —, placed at the head of the left hand column, in which the argument is in this case found, shows to be subtractive. It is plain also, from the figure, that it should be subtractive; for, in adding the equation of the centre, (17,) we added twice the whole angle EMC, which was too much by twice the angle HMC. We now correct the error, by subtracting the equation just found from the longitude and anomaly previously obtained, (34,) which leaves for the former 66°.8610, and for the latter, 144°.291.

61. All the remarks that were made in article 34 on the subject of variation, will apply also to evection, since both are caused by the sun's disturbing influence. The method of taking the requisite correction from the table (table 15) is also the same, only that in this case, the argument for *evection*, viz., the moon's mean anomaly diminished by twice the excess of the moon's longitude over that of the sun, is to be sought for at the top or bottom of the table, instead of the argument for *variation.*

The correction, as found in the table, is —.0100, but since one argument is found in the *inner* gnomon, at the bottom, and the other in the *outer* one, at the right, the sign is to be changed, (34,) and the correction becomes +.0100, which, added to the longitude and anomaly last found, makes them respectively 67°.8710, and 144°.301.

* This is the moon's mean anomaly, corrected by the annual equation of its perigee, which it is proper to do, because that inequality affects only the average progressive motion of the perigee at different seasons of the year, and is in no way connected with that which we are now considering, or any other which has reference to the position of the moon in its orbit.

CHAPTER VII.

NODAL EQUATION OF THE MOON'S LONGITUDE, AND REDUCTION TO THE ECLIPTIC.

62. In the numerous corrections that we have had occasion to apply to the moon's longitude and anomaly, growing out of the disturbing influence of the sun, the orbits of both have been supposed to lie in the same plane; or the latter to lie in the plane of the orbit of the former. The first of these suppositions is never true, and the latter only twice in a year; viz., when the sun passes the moon's nodes. At all other times, it is either on one side of the plane of the moon's orbit or the other. Now it is evident that the sun's disturbing influence, in the various ways we have been speaking of, must be less than if it lay in the plane of the moon's orbit; for, in order to make our reasoning good, its attraction must be resolved into two forces, one lying in the plane of the orbit, and the other at right angles to it, which necessarily creates a loss of force. If it were always at a fixed mean distance from the plane, a proper allowance might be made in computing the inequalities, and the work would thus be accurate without further correction. In fact, the quantities in the tables which we have been using, were calculated on that supposition. But since the distance is variable, additional corrections are necessary for all that we have applied in the three preceding chapters. We will select, as an example, the annual equation of the moon's longitude, discussed in chapter 4th, remarking, as we pass, that if we were to attempt to apply all the corrections resulting from causes like that under consideration, and from the effect of one correction in altering the argument from which others had been obtained, our task would be endless. The business is, at best, only a series of approximations.

63. When the sun is passing one of the moon's nodes, being in the plane of the orbit, its attractive force exceeds the mean, so far as the circumstance now under consideration is concerned, dilating the moon's orbit (19) and increasing the periodic time more than usual. The moon must therefore fall behind its mean place, and continue to do so more and more, till the sun reaches its point of mean distance, about 45° from the node. As the sun continues to recede from the plane of the moon's orbit, its disturbing influence

must grow less, allowing the moon to contract its orbit and shorten its periodic time, till finally, when the former is 90° from the node, the latter will have gained up what it had lost, so that its mean and true place will again coincide.

The reverse of all this will take place when the sun is in the next quadrant. Its disturbing influence being a minimum at the outset, the moon must get ahead of its mean place; and it will not lose what it thus gains till the sun reaches the next node. Hence, if the sun's longitude exceeds that of one of the moon's nodes by less than 90°, something must be subtracted from the longitude and anomaly, as already obtained; but added, if the excess is greater than 90°. Or, reckoning from the ascending node, a subtractive equation must be applied in the 1st and 3d quadrants, and an additive one in the 2d and 4th. Such an equation is termed the Nodal Equation of the moon's longitude.*

64. To find how far the sun is from the node, the longitude of the latter (13) must be subtracted from that of the former, (17.) Entering table 16th with the argument thus found, viz., 4°.3486, in the same manner as we did table 6th and others, we find the equation to be .0026, which the sign — at the head of the column containing the argument, as well as our previous reasoning, shows must be subtractive. The resulting longitude of the moon becomes 66°.8684, and the anomaly 144°.298.

65. The moon's anomaly is now altered considerably, by reason of the various equations that have been applied to it, from what it was when we used it to take out the equation of the centre, in article 17th; and since this equation is a very important one, our work will be more accurate if we now take it out again, and by whatever amount it differs from what it was as first taken out, correct the moon's longitude. The anomaly, as used in article 17th, was 145°.078, which gave as an equation +3°.4154, while now it is but 144°.298, which gives for the equation +3°.4834, so that we did not add enough to the moon's longitade by 0°.0680. Adding this now, we have 66°.0364, which may be regarded as the true longitude of the moon, reckoned on its orbit, or, as it is usually termed, the true Orbit Longitude.

* I find no name for this equation in any treatise on astronomy that I have met with, and have given it one that seems to be indicative of its character.

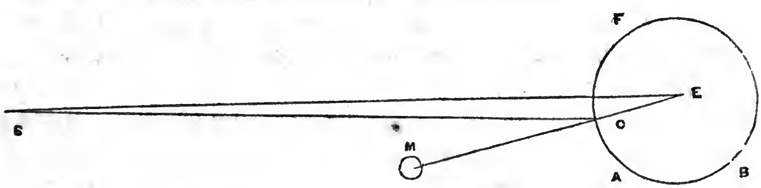
66. The plane of the lunar orbit being inclined to that of the ecliptic, causes longitudes reckoned on it to be different from what they would be if reckoned on the ecliptic. And since the longitudes of the heavenly bodies are referred to the latter, the *orbit longitude* just found needs one more correction to reduce it to the ecliptic. The argument, found by subtracting the longitude of the moon's node from that of the moon itself, is 5°.9356, and the corresponding equation, obtained from table 17th, is —.0233. This applied, leaves 66°.9131 for the moon's true longitude from the mean vernal equinox.

CHAPTER VIII.

LUNAR, OR MENSTRUAL EQUATION OF THE SUN'S LONGITUDE AND NUTATION.

67. It was observed in article 2d, that any motion or change of motion in the earth, produced apparently a precisely similar one in the sun. Now, the earth, like the moon, revolves round the common centre of gravity of the two, and is, therefore, subject to inequalities in this motion, the same in kind as those we have been considering in that of the moon, though far less in degree, owing to the earth's greater weight, and consequently close proximity to the centre of gravity. These inequalities, small in themselves, are rendered vastly smaller in their effect upon the sun's apparent motion, by reason of the great distance of the latter.

Fig. 14.



Let S (Fig. 14) represent the sun, ABF the earth, E its centre, M the moon, and C the common centre of gravity between the earth and moon, about which both revolve. The distance from E

to C is not far from 2970 miles, or about three-fourths of the earth's radius.

It is manifest that the longitude of the sun, as seen from E, will differ from its longitude as seen from C, by the angle CSE. When the angle MES is either 0° or 180°, the angle CSE will disappear, and when it is of any other size, the latter angle can be calculated; for, in the triangle CES, the two sides, CS and CE, and the angle CES are known. We assume here, that E revolves in a circle round C, keeping CE of uniform length. It is plain from the diagram, that if the longitude of the moon exceeds that of the sun, the latter will be increased by the angle CSE; but the contrary, if the longitude of the sun is greatest. In other words, if the longitude of the moon, diminished by that of the sun, is less than 180°, the equation will be additive, but if greater, subtractive.

68. The longitude of the sun, as found in chapter 3d, is 65°.3494, and that of the moon, as finally corrected, (67,) 66°.9131. The excess of the latter above the former is 1°.5637. Entering table 18th with this number, in the usual way, we find that, in the present case, the correction is inappreciable, unless we extend our decimals further; so that the longitude obtained in chapter 3d is to be considered correct.

69. The error occasioned by regarding the orbit of the earth's centre as a circle instead of an elipse, as it in fact is, might be corrected by introducing an equation corresponding to the equation of the centre of the sun or moon. But it would never be necessary, unless extreme accuracy were required; for the whole correction which we just undertook to apply, when a maximum, is but the decimals of a degree, .0021; and since the eccentricity of the orbit amounts to hardly more than 1-20 of the radius, the error can never be more than about .0001, which is less than half a second. Much less then must the *inequalities* in the motion be appreciable.

CHAPTER IX.

NUTATION IN LONGITUDE.

70. THE equinoxes are not stationary, but move slowly westward, which necessarily affects the longitudes of all the heavenly bodies, since they are reckoned from the vernal equinox. If the rate of motion were uniform, the longitudes of the sun, moon, and moon's nodes, which we have obtained in the preceding chapters, would nevertheless be correct; for the tables from which we obtained the mean longitudes in article 11th, are based upon the supposition of a uniform rate of precession of the equinoxes, and allowance is consequently made for it. But it is not uniform, and we are now to look into the causes of the inequality, and make the requisite correction in the longitudes on account of it.

71. If a body were to revolve round the earth in an orbit not coinciding with the ecliptic, so that it would be sometimes north and sometimes south of the plane of the latter, we can see that whenever it were thus situated, the sun's attraction must tend to draw it back into the aforesaid plane. To illustrate by a diagram, let SD (Fig. 15) represent the plane of the ecliptic, and MM′ that of the revolving body, both seen edgewise, S the sun, and E the earth.

Fig. 15.

When the body is at M the sun's attration on it, in the direction SM, may be resolved into two other forces, in the directions SC and CM, the latter of which tends to draw the body directly into the plane SD. In like manner, when the body is at M′, it is drawn toward the plane SD, by a force represented by M′D; and so of any other point out of the plane of the ecliptic. The consequence is, that the body, as it revolves round its orbit, which we will suppose it to do in an easterly direction, is drawn into the plane of the ecliptic, and made to cross it sooner, that is, farther westward, every succeeding revolution.* The same would be true of any number of bodies similarly situated, so that our reasoning will be good, even if they were multiplied to such a

* The reader will perceive, that the condition of our supposed body corresponds, in every respect, with that of the moon revolving round the earth, and hence will see the cause of the retrograde motion of the moon's nodes.

degree as to form a continuous ring entirely round the orbit. Nor will it alter the principle, if we suppose the bodies, or ring, very near to the earth, or even attached to it, only that, in the latter case, they would communicate their motion to the earth, and so could not move without dragging the earth with them, which would greatly deaden any motion, or change of motion they would otherwise have.

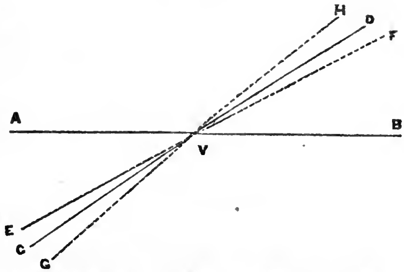
Now, from the spheroidal form of the earth, there is a protruberant mass of matter about eighteen miles thick girdling its equator, every particle of which is situated precisely as we supposed our imaginary bodies or ring to be. The effect we have described must therefore follow, and every place—as Quito, for example—must every day, by the diurnal motion of the earth, cross the ecliptic at a point farther west than on the day previous. Or, to illustrate farther; suppose the plane of the ecliptic to be a vast sheet of some material substance, with an orifice of sufficient size to admit the earth, and to allow it to revolve freely on its axis. Now, if a man were stationed on Mount Chimborazo, (which we will suppose to be on the equator, though it is not precisely so,) and every time the earth rolled round, so as to carry him under the plane, which would be every twelve hours, should mark on the edge the place under which he passed it, these marks would be continually farther and farther west by about fifteen feet.

72. The attraction of the moon also conspires with that of the sun in causing a precession of the equinoxes; for the plane of its orbit being much more nearly coincident with the ecliptic than that of the equator is, it may be regarded as another body lying in the plane of the ecliptic, and conspiring with the sun in its influence upon the earth.

73. It is evident from Fig. 15th, that the greater the inclination of the planes SD and MM′, the greater must be the forces represented by CM and DM′, and consequently the more rapid must be the retrograde motion of the points of intersection. So far, therefore, as the moon's influence is concerned, the greater the obliquity of its orbit to the equator, the more rapid must be the precession of the equinoxes.

Let CD (Fig. 16) represent a portion of the ecliptic, seen edgewise, V the vernal equinox, and EF and GH portions of the moon's orbit, making an angle of about 5° with the ecliptic; GH representing it when the ascending node is at V, and EF when the ascending node is there. Also, let AB represent a portion of the equator, making an angle of about $23\frac{1}{2}°$ with the ecliptic. Then HVB, the inclination of the moon's orbit to the equator, when the ascending node is at V, equals about $28\frac{1}{2}°$; and FVB, the inclination when the ascending node is there, equals about $18\frac{1}{2}°$.

Fig. 16.



74. When, therefore, the ascending node, in its retrograde course, passes the vernal equinox, which it does once in about nineteen years, the rate of precession must considerably exceed the mean, and the equinoxes must immediately get too far west, which would increase the longitude af all the heavenly bodies. The same would be true all the while that the node was slowly working its way backward round to the autumnal equinox; for though the rate of precession would continually diminish, and become a mean when the node was 90° back, or west of the vernal equinox, yet it would take the whole of the next quadrant for it to lose what it had gained in the first. Thus, when the ascending node gets round to the autumnal equinox, which would bring the descending node to the vernal, all the longitudes would become right, or in their mean state again. But the rate of precession is now a minimum, and on principles similar to those we have been discussing, it is apparent, that, while the node is passing through the other half of its orbit, the longitude of all the heavenly bodies must be less than the mean. Thus, for a period of about nine and a half years, all longitudes are greater than the mean, and then, for the same period, less; and so on, alternately. This is called Lunar Nutation in Longitude.

75. It is plain, that when the ascending node is passing from the vernal back to the autumnal equinox, its longitude must exceed 180°, and be less than 180° when it is in the other half of its orbit; so that we can know by the longitude of the node, whether to add to or subtract from our mean longitudes.

76. At the time for which we are calculating, the mean longitude of the ascending node (11) is 61°0883; and entering table 19th with this argument, we find the amount to be subtracted from the longitudes of all the heavenly bodies at that time, is .0042, which will leave us for the moon's longitude (66) 66°.9089; for the sun's (18) 65°.3452, and for the moon's node (13) 60°.9966.

77. There is another inequality in the rate of the Precession of the Equinoxes, called Solar Nutation, and occasioned by the variable distance of the sun from the plane of the equator in the course of a year. But it is so small (never amounting to much over one second) that it may be disregarded without material error.

CHAPTER X.

TRUE TIME LONGITUDES AND ANOMALIES.

78. By comparing the true longitudes of the sun and moon, found in the last chapter, we find that the latter is greatest by 1°.5637, which shows that the moon has passed by the sun, and that the eclipse is over. It remains (12) for us to subtract such an amount from the mean time of new moon (11) as it must have taken the moon to gain this difference, and in order to do so, we must know the relative velocities of the sun and moon in their orbits at the time.

Their motions are swiftest in perigee, and grow slower as they recede from it; hence their anomalies are the proper arguments for determining their motions. We may therefore enter tables 20th and 21st, with the anomalies of the sun and moon respectively as arguments, and take out their hourly motions. The former we find to be .0401, and the latter .5018.

All the other inequalities treated of in the preceding chapters, must likewise affect the moon's hourly motion, of which Variation and Evection are the most important. The effect of Variation, as is plain from the theory, is to increase the moon's velocity in syzygy and diminish it in quadrature. Now, in an eclipse, the moon is always in syzygy, and hence we must add to its hourly motion the quantity given in the margin of table 21st. To correct the

moon's hourly motion for Evection, we must enter table 22d with the same argument that was used for that inequality in article 60th, and in the middle column we find the equation, which is to be applied to the hourly motion according to its sign.

The following is the operation in the case before us:—

Moon's hourly motion, by table 21st, - -	.5018
Add for Variation, - - - - - -	.0115
	.5133
Subtract for Evection, by table 22d, - -	.0092
	.5041
Subtract sun's hourly motion, - - -	.0401
Hourly gain of the moon upon the sun, - -	.4640

Now, by simple proportion, we can find how long it must have taken the moon to gain the difference in the longitudes of the sun and moon, viz. 1°.5637. Thus,

.4640 : 1 hour : : 1°.5637 : the time required, which is thus found to be 3 hours, 22 minutes, and 12 seconds. This subtracted from the time of mean new moon, found in article 11th, leaves for the true time of new moon in May, 26d. 8h. 48m. 44sec.

79. The time thus found is Greenwich time, and to reduce it to that of any other place, allowance must be made for the difference of longitude, viz., 4 minutes of time for each degree of longitude. It is also to be observed, that the astronomical day begins at noon, and counts the 24 hours round to the next noon.

80. The longitudes and anomalies of the sun and moon must now be corrected, by subtracting their motions during the correction just applied to the time. If that correction had been additive, this would be so also. The amount can be easily found from their hourly motions, thus—

One hour : the moon's hourly motion, viz., .5041 : : 3h. 22m. 12 sec. : the correction required in its longitude and anomaly, which is thus found to be 1°.6988.

One hour : the sun's hourly motion, viz., .0401 : : 3h. 22m. 12 sec. : the correction required in its longitude and anomaly, which is thus found to be 0°.1351.

At the same time the longitude of the node must be corrected, by taking from table 4th its motion during the same time, and applying it, with the contrary sign from that of the other motions, because it moves in the opposite direction.

81. The reader will get a clearer idea of the process of calculating the time of an eclipse, if we now give a synopsis of the work that we have been through in the foregoing chapters.

EXAMPLE.

Showing the method of calculating the time of a Solar Eclipse.

	Time.	Sun's Anomaly.	Sun's Longitude.	Moon's Anomaly.	Moon's Longitude.	Longitude of Node.
	d. h. m. s.	°	°	°	°	°
Mean new moon, March, 1854	28 10 42 50	85.586	6.0194	93.665	6.0194	64.2158
Add two lunations,............	59 1 28 6	58.211	58.2135	51.634	58.2135	-3.1275
Mean new moon in May,......	26 12 10 56	143.797	64.2329	145.299	64.2329	61.0883
An. equa'n moon's per. & node	....	....	....	—.221	....	—.0875
				145.078		61.0008
Equation of the centre,........	....	....	+1.1165	....	+3.4154	
			65.3494	145.078	67.6483	
An'l equation of moon's long.	....	....	....	—.112	—.1118	
				144.966	67.5365	
Secular equa. of moon's long.,	....	....	....	+.001	+.0009	
				144.967	67.5374	
Variation,......................	....	....	....	+.045	+.0445	
				145.012	67.5819	
Annual equation of variation,.	....	....	....	—.005	—.0053	
				145.007	67.5766	
Evection,......................	....	....	....	—.716	—.7156	
				144.291	66.8610	
Annual equation of evection,.	....	....	....	+.010	+.0100	
				144.301	66.8710	
Nodal equation of the moon's longitude, ...	....	....	....	—.003	—.0026	
				144.298	66.8684	
Corrrection of the equation of the centre, ...	....	....	....	....	+.0680	
					66.9364	
Reduction to the ecliptic,......	....	....	....	....	—.0233	
					66.9131	
Lunar nutation,..................	....	....	—.0042	....	—.0042	—.0042
			65.3452		66.9089	60.9966
Correct for difference in long. of sun & moon, ...	3 22 12	—.135	—.1351	—1.699	—1.6988	+.0074
True time, long's & anomalies,	26 8 48 44	143.662	65.2101	142.599	65.2101	61.0040

82. It can hardly fail to suggest itself to the attentive reader, that so great an alteration in the time, as that which was made in article 78th, must in some degree vitiate the result of our work. The anomalies, and nearly all the arguments for the inequalities would vary in the interim. It would hence seem desirable to have obtained, if possible, some nearer approximation to the true time at the outset. The "preliminary equations"* in tables 27 and 28 are designed for this purpose, but the theory of them could not be well explained at that stage of our work, where it was necessary to introduce them, if at all.

The construction of these tables is as follows. Since at the time of new or full moon, the argument for evection is the same as that for the equation of the moon's centre, viz., the moon's mean anomaly, the separate effects of the two on the time are united in the first preliminary equation. Also, since both the equation of the sun's centre, and the annual equation of the moon's longitude, depend on the sun's anomaly, they are united in like manner in the second preliminary equation.

After having found the time of mean new or full moon, as described in article 11th, we apply to it these equations, and then take from table 4th the mean motions in longitude and anomaly during the time so applied. These motions applied to the mean longitudes and anomalies at the mean time, give the mean longitudes and anomalies at the corrected time; and we then proceed to calculate the true longitudes at the corrected time, in the same manner as we have done for the mean time in the foregoing chapters. We will illustrate, by example, the method of using these tables, and at the same time show how to calculate a lunar eclipse.

83. It will not be necessary to give much more than a synopsis of the operation, as the time of a lunar eclipse is calculated precisely in the same manner as one of the sun, only that the half lunation in table 3d, is used in order to give the time of mean *full* moon, and the longitudes of the sun and moon, instead of being made to agree, are made to differ just 180°.

Let us inquire whether there will be an eclipse of the moon when the sun passes its ascending node in the year 1844.

Turning to table 2d, we find that the longitude of the ascending

* These are the same as those usually termed 1st and 2d equations of the mean to the true syzygy.

node in March of that year is 258°, and we therefore (11) take from table 3d such a number of lunations as, added to the half lunation at the foot of the table, will contain 258 days, or thereabouts. In 8½ lunations there are 251 days, which is the nearest to 258 that we can get from the table. This will carry the time forward to November, and will give us for the longitude of the sun 244°, and of the node about 245°. The sun will, therefore, be but 1° from the node, and so far within the lunar ecliptic limit (7) that there cannot fail to be an eclipse of considerable size. We will proceed to calculate it. From tables 2d, 3d and 5th, we obtain the following:—

	Time.	Sun's Anomaly.	Sun's Longitude.	Moon's Anomaly.	Moon's Longitude.	Longitude of Node.
	d. h. m. s.	°	°	°	°	°
Mean new moon in March, 1844,	18 15 40 54	76.522	356.7831	132.366	356.7831	253.1245
Add eight lunations,....................	246 5 52 23	232.843	232.8530	206.535	232.8530	—12.5102
Add half a lunation,....................	14 18 22 1	14.553	14.5534	192.908	194.5534	-- .7819
Mean full moon in November,...	24 15 55 18	323.918	244.1895	171.809	64.1895	244.8324

Entering tables 27 and 28, with the anomalies of the moon and sun respectively as arguments, and making the proper proportions, we obtian the "preliminary equations," the first of which is 1h. 30m. 1sec., subtractive, and the second 2h. 30m. 19sec., also subtractive. The sum of the two is 4h. 0m. 20sec., which must be subtracted from the mean time found above. Also the motions of the sun and moon during this time, both in longitude and anomaly, must be subtracted, and that of the node added, because it moves the other way. The quantities in table 4th, from which we obtain these motions, are given only for the units of the arguments, but will answer just as well for tens, by removing the decimal point one place to the right. Thus the sun's motion in anomaly, according to the table, is, for 2 days, 1°.9712, and for 20 days 19°.712. When used for units, the right hand figure may be omitted, and the next reckoned according to its nearest value. The following is the operation for finding the motions in the case before us:—

	Sun's Anomaly.	Sun's Longitude.	Moon's Anomaly.	Moon's Longitude.	Longitude of Node.
Motion in 4 hours,..................................	0°.164	0°.1643	2°.177	2°.1961	0°.0088
Do. in 20 seconds,..............................	0°.000	0°.0002	0°.003	0°.0030	0°.0000
Total motion in 4h. 0m. 20sec.,................	0°.164	0°.1645	2°.180	2°.1991	0°.0088

84. After applying these corrections to the time, longitudes and anomalies, the process of calculation is, throughout, the same as for a solar eclipse, with the exception already mentioned;* and it is unnecessary to go through it in detail. The following is a synopsis of the calculation :—

EXAMPLE.

Showing the method of calculating the time of a Lunar Eclipse.

	Time.	Sun's Anomaly.	Sun's Longitude.	Moon's Anomaly.	Moon's Longitude.	Longitude of Node.
	d. h. m. s.	°	°	°	°	°
Mean new moon in March, 1844,........	18 15 40 54	76.522	356.7831	132.366	356.7831	258.1245
Add eight lunations,....................	136 5 52 23	232.843	232.8530	206.535	232.8530	−12.5102
Add half a lunation,....................	14 18 22 1	14.553	14.5534	192.908	194.5530	— .7819
Mean full moon in November,.........	24 15 55 18	323.918	244.1895	171.809	64.1895	244.8324
1st cor. in time, with cor'pond'g motions,	— 4 0 20	— .164	—.1645	—2.180	—2.1991	+.0088
	24 11 54 58	323.754	244.0250	169.629	61.9904	244.8412
Annual equation of moon's per. and node,	....	....	....	+ .217	....	+.0897
				169.846		244.9309
Equation of the centre,................	....	....	—1.1560	....	+1.0412	
			242.8690		63.0316	
Annual equation of moon's longitude,..	....	....	....	+ .109	+.1087	
				169.955	63.1403	
Secular equation of moon's longitude,..	....	....	....	+ .001	+.0006	
				169.956	63.1409	
Variation,..............................	....	....	....	+ .006	+.0058	
				169.962	63.1467	
Annual equation of variation,...........	....	....	....	+ .003	+.0034	
				169.965	63.1501	
Evection,..............................	....	....	....	— .193	—.1930	
				169.772	62.9571	
Annual equation of evection,..........	....	....	....	+ .007	+.0068	
				169.779	62.9639	
Nodal equation of moon's longitude,....	....	....	....	+ .001	+.0013	
				169.780	62.9652	
Correction of the equation of the centre,	....	....	....	....	+.0068	
					62.9720	
Reduction to the ecliptic,..............	....	....	....	....	+.0077	
					62.9797	
Lunar nutation,........................	....	....	+ .0044	....	+.0044	+.0044
			242.8734		62.9841	244.9353
Cor. for difference* in lon. of sun & moon,	— 14 41	— .010	— .0103	— .121	—.1210	+.0004
True time, longitudes and anomalies,....	24 11 40 17	323.744	242.8631	169.659	62.8631	244.9357

* In lunar eclipses it is this difference, + or —180°.

CHAPTER XI.

ELEMENTS OF AN ECLIPSE.

85. The following elements or data, are all that are needed for making any calculation that we may desire in regard to an eclipse, either solar or lunar; such as the place on the earth's surface, where the sun will be centrally eclipsed at any given time, while the eclipse lasts; the portions of the earth where an eclipse will be visible, and the time when it will commence, become a maximum, and terminate; or the size of an eclipse, at any given place and time. The 10th is not needed in solar eclipses, nor the 2d, 3d, and 12th in lunar.

1st. The time of new or full moon.

2d. The longitudes of the sun and moon.

3d. The obliquity of the ecliptic to the equator.

4th. The moon's latitude.

5th. The sun's hourly motion.

6th. The moon's relative hourly motion, or the excess of its hourly motion over that of the sun.

7th. The sun's apparent semidiameter as seen from the earth.

8th. The moon's do.

9th. The apparent semidiameter of the earth as seen from the moon, which is the same as the moon's horizontal parallax.

10th. The apparent semidiameter of the earth's shadow where it eclipses the moon, as seen from the earth.

11th. The angle of the moon's visible path with the ecliptic.

12th. The sun's declination.

86. The method of obtaining the first two, was explained in the last chapter.

87. The mean obliquity of the ecliptic to the equator in the year 1840, was 24° 27′ 62″.52, but it decreases at the rate of about half a second a year, owing to the attraction of the planets. It is also subject to an inequality, whose period is about 19 years, depending on the longitude of the moon's node: for it is evident from the theory of lunar nutation, that the moon's influence must affect the

obliquity of the equator to the ecliptic, as well as the place of their points of intersection. Table 23d gives the obliquity on the 1st of January in each year of the present century after 1840, taking both these causes into account, from which it can be readily found for any time in the year by inspection. In January, 1854, it is 23° 27′ 33″.6, and in January, 1855, it is 23° 27′ 35″.7. Hence, at the time of our solar eclipse in May, 1854, it is 23° 27′ 34″.5.

88. The moon's latitude depends on its distance from the node, and the inclination of the plane of its orbit; but the inclination, and also the place of the node, varies according to the situation of the sun in respect to the node. Hence there are two principal equations of the moon's latitude, one depending on the distance of the moon from the node, and the other on that of the sun. Now in eclipses, the distance of both luminaries from the node is the same, so that the two equations may be combined into one, which is done in table 24th. The argument is found by subtracting the longitude of the node from that of the sun and moon, which leaves, in our solar eclipse, 4°.2061, and in the lunar, 177°.9274. The moon's latitude, as determined by these arguments, is, in the former case, the decimals of a degree .3664, or a little over one-third of a degree, and in the latter, .1810. It is to be noticed, that in table 24th the figures of the argument at the right and left are whole degrees, and those at the top and bottom the first place of decimals.

It must be readily seen, that the moon's latitude must be north for the first 180° after it leaves the ascending node; and that it moves northerly, or *ascends*, through the first quadrant, and southerly, or *descends*, through the second: also, that in the other half of the orbit, its latitude must be south, being *descending* in the first quadrant, and *ascending* in the second. These facts are indicated in the table by capital letters at the head of the columns containing the argument.

89. The method of finding the 5th and 6th elements was explained in the last chapter, (78;) but if much accuracy were required, they would have to be now computed over again for our solar eclipse, because the anomalies have been changed. Calculated from the anomalies as finally corrected, in the same manner as was done in article 78, the hourly motion of the sun in our solar

eclipse we find to be .0401, and in the lunar .0421; and the relative hourly motion of the moon is .4649 in the solar eclipse, and .4523 in the lunar.

90. The 7th element is obtained from the 1st column of table 26th, where it is sufficiently explained.

91. Table 21st, columns 1st and 3d, give the 8th and 9th elements, so far as they depend on the elliptical form of the moon's orbit. But the effect of Variation is to throw the orbit into a kind of oval, with its shortest diameter lying in syzygy. From this cause, the distance of the moon from the earth is less when it is new or full than at other times, which must increase their apparent size as viewed from each other. Hence the corrections in the margin of table 21st. Also Evection, by altering the shape of the moon's orbit, affects its apparent size and parallax, so that a farther correction becomes necessary from table 22d. The following shows the method of obtaining these elements for the eclipse of May, 1854:—

	Semidiameter.	Parallax.
Values taken from table 21st,............	.2482	.9097
Variation,............	+.0020	+.0073
	.2502	.9170
Evection,............	—.0023	—.0080
True semidiameter and parallax,............	.2479	.9090

The semidiameter and parallax at the time of the lunar eclipse in November, 1844, obtained in the same way, are .2453 and .8989 respectively.

92. It was shown in article 4th, that our 10th element, viz., the apparent semidiameter of the section of the earth's shadow that eclipses the moon, is equal to the sum of the parallaxes of the sun and moon, diminished by the sun's apparent semidiameter. The sun's parallax may always be put down at .0024, and the methods of obtaining the other data for finding this element have been already explained. Thus in the lunar eclipse which we have taken as an example,

The sun's parallax is - - - - - -	.0024
The moon's do., as just found, is - - - -	.8989
	.9013
The sun's semidiameter (see 7th element) is - -	.2707
The semidiameter of earth's shadow is - - -	.6306

93. The angle which the moon's path makes with the ecliptic varies according to the moon's distance from the node. When at the node, it makes the same angle as the planes of the two orbits; but when it is 90° from it, its motion becomes parallel to the ecliptic. But the inclination of the planes also varies, depending, as was remarked above, on the distance of the sun from the node. The two influences may, however, be combined into one at the time of an eclipse, in the same manner as in table 24th. And not only does the *real* angle vary from both these causes, but the *apparent* angle is increased by the earth's motion in the same direction; and since the rate of the latter, as compared with the moon's motion, is quite variable, the apparent angle must vary also from this cause. All these causes are taken into account in table 25th. This table has two arguments, viz., 1st, the difference between the hourly motions of the sun and moon, (for the motion of the earth is measured by the apparent motion of the sun,) the first two decimal places of which are placed at the top; and 2d, the distance of the sun or moon from the node, which is placed at the right and left. In the solar eclipse we are calculating, the former (89) is .4649, and the latter (found by subtracting the longitude of the node from that of the moon) 4°.2061. These give the angle 5° 44′ 33″, ascending. In the same way the angle at the time of our lunar eclipse is found to be 5° 45′ 41″ descending.

94. The sun's declination, which is our 12th element, can easily be computed from its longitude by spherical trigonometry, since the obliquity of the ecliptic is known, (87;) for its longitude, right ascension, and declination form a right-angled spherical triangle, of which an angle and one side is known. Table 26th gives the declination calculated from the obliquity in the year 1840, which is sufficiently exact for our purpose, though, after a lapse of years, it must evidently need correction. Entering this table, with the sun's longitude at the time of our solar eclipse as an argument, we obtain the declination 21°.1871.

Since the sun starts northerly from the vernal equinox, its declination must be north for the first 180°, and south through the rest of the orbit. This fact is indicated by the capital letters at the head of the columns of the argument.

95. The elements collected are as follows:—

	Solar Eclipse.	Lunar Eclipse.
	d. h. m. s.	d. h. m. s.
1. True time of the eclipse,..................	May 26 8 48 44	Nov. 24 11 40 17
2. Longitude of the sun and moon,.......	65°.2101	
3. Obliquity of ecliptic to equator,........	23° 27′ 34″.5	
4. Moon's latitude,..........................	0°.3664 (north)	0°.1810 (north)
5. Sun's hourly motion, ...	0°.0401................	0°.0421
6. Moon's relative do........................	0°.4649................	0°.4523
7. Sun's apparent semidiameter,..........	0°.2635................	0°.2707
8. Moon's do................................	0°.2479................	0°.2453
9. Moon's horizontal parallax,.............	0°.9090................	0°.8989
10. Semidiameter of earth's shadow,......	....	0°.6306
11. Angle of moon's visible path with eclip.	5° 44′ 33″ (ascend.)	5° 45′ 41″ (descend.)
12. Sun's declination,........................	21°.1871 (north)	

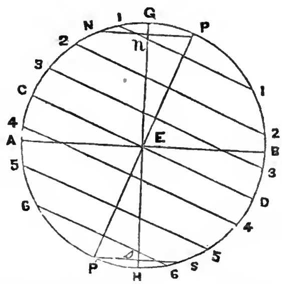
CHAPTER XII.

DELINEATION OF A SOLAR ECLIPSE.

96. To find whether a solar eclipse will be visible at a particular place, and if so, its size and general appearance there, it is more convenient to first reverse the order of viewing the phenomenon, and to suppose the spectator placed at the centre of the sun, to look down upon the earth, and see the moon passing across its disc. From so vast a distance, the earth would appear to him, as the sun does to us, like a flat circular disc. The circle of illumination would be to our observer the circle of the disc, and all circles whose planes were perpendicular to this would be seen edgewise, and would appear to him like straight lines. Their arcs would seem to be only of the length of the straight lines that they would subtend, as viewed by him. Such circles as were seen obliquely, would appear elliptical in their shape. Let us suppose him to take

his station when the sun is in the vernal equinox, about the 21st of March, and to retain it for a year, accompanying the sun in its apparent annual round. Being always in the plane of the ecliptic, it would appear to him like a straight line dividing the earth into two equal parts, one half lying north and the other south of it. At first also, being in the plane of the equator, it too would be seen as a straight line, as well as all the parallels of latitude; yet not parallel with the ecliptic. The west end of the equator would be north of the ecliptic and the east end south, crossing it in the centre at an angle of about $23\frac{1}{2}°$, as in Fig. 17, where AB represents the ecliptic, CD the equator, PP′ the earth's axis, P and P′ its poles, GH the axis of the ecliptic, and 1 1, 2 2, 3 3, &c., parallels of latitude.

Fig. 17.



As the sun advances, it gets north of the plane of the equator and of the parallels of latitude, and they will no longer appear as straight lines, but will seem bent downward toward the south. The earth's axis will approach to parallelism with that of the ecliptic; and the poles revolving in circles whose planes are parallel to that of the ecliptic, will seem to move in straight lines toward *n* and *s* till on the 20th of June, or thereabouts, when the sun reaches the summer solstice, the two axes will coincide, and the north pole will be seen at *n*. The south pole will be invisible, being hid behind a segment of the earth; but if the earth were transparent it would appear at *s*.

97. The sun still advancing, the earth's axis will appear again on the other side of GH, the poles will approach toward N and S, and the parallels of latitude will become less curved. And when the sun reaches the autumnal equinox, in September, the latter will again become straight lines, but lying the opposite way from what they did in March, and the poles will appear at N and S.

98. As soon as the sun has passed the autumnal equinox, the parallels of latitude will appear curved again, but upward toward the north, instead of downward, because the sun will now be on the south side of their planes. The poles will recede along the

lines NP and SP′, passing n and s when the sun is at the winter solstice, in December, and finally arriving at P and P′ about the 21st of March, when everything assumes the same aspect as when he started.

99. A common terrestrial globe will serve to present these various appearances to the reader's view, in a much clearer light than can be done by any verbal description. Let the north pole be elevated about 66½° above the point marked "North," on the wooden horizon, and then the latter will represent the ecliptic. If now the globe be placed upon a table in the centre of the room, so that the wooden horizon may be on a level with the eye, and the reader, after having found the 21st of March on the horizon, should retire across the room in that direction, he will see the globe precisely as represented in Fig. 17. Let him now pass slowly round the room in the order of the months on the horizon, and all the appearances we have described will be presented to his view.

100. When he is in the direction marked May 26, he will have a true representation of the earth, as it would appear to our observer at the sun, at the time of the solar eclipse we have been calculating. We will endeavor to represent the same by a figure, and also the appearance of the moon passing over the earth's disc. In order to give proper proportions to the several parts of our figure, we must be able to mark down their relative dimensions. In common plans and drawings these are given in miles, feet, inches, or some other direct measure of length; but in astronomy it is found more convenient to determine them by the angle which they would subtend when viewed from a given distance, as we did in the last chapter. And this answers the purpose just as well, provided the distance be sufficiently great, for the apparent would be very nearly proportional to the real size of the object. The dimensions which respect the moon and earth, given in the last chapter, are the angles that the objects would subtend at a distance of about 237,000 miles, or the distance between the earth and moon. The reader must not here fall into the error of supposing that our observer has changed his position. He is still at the sun, and these angles are given merely for the purpose of determining the relative sizes of the objects that we wish to draw.

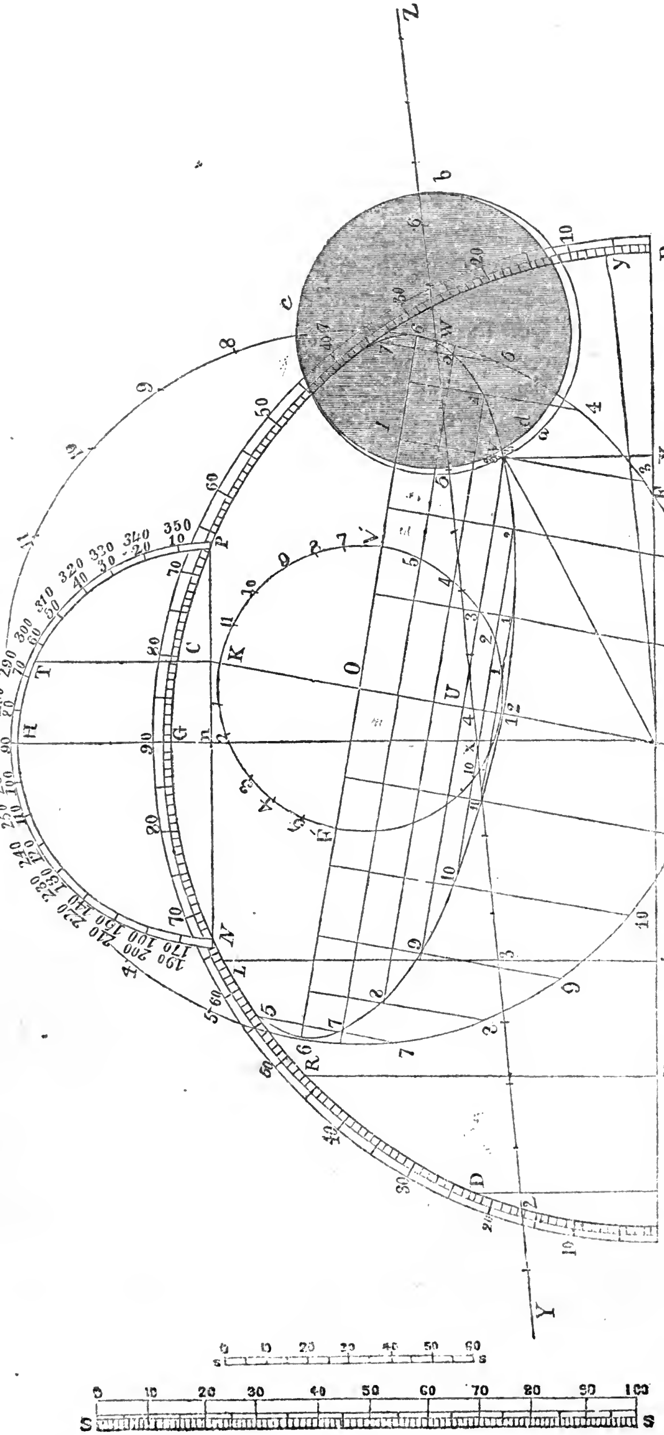


Fig. 18.—SOLAR ECLIPSE, MAY, 1854.

101. Our first business is to make a scale SS (Fig. 18) of any convenient length, and divide it into 100 equal parts. This scale may be considered to be of such length, that if seen perpendicularly at the distance of the moon from the earth, it would subtend an angle of one degree, and of course each of the parts would subtend .01 of a degree. The earth's semidiameter (95) seen at that distance, subtends an angle of 0°.9090. If therefore, with this distance, taken from our scale, as radius, (counting the two first decimals places, with a proper proportion for the two last,) we describe the semicircle AGB, the line AB may represent that portion of the plane of the ecliptic which intersects the earth, and the whole semicircle AGB, the half of the earth's disc that is seen north of it. Or, which is the same thing, AB may represent the wooden horizon of the globe adjusted as above described, and AGB the half of the globe that is above it.

102. We will next find the position of the north pole of the earth, i. e., the point on the disc where it would be seen. A glance at the globe will show that it is not at the top, nor anywhere in the circumference of the disc. In fact, we have shown (96) that it must appear to move in the right line PN, leaving P about the 21st of March, and arriving at *n* about the 20th of June. Hence, at the time of our eclipse, it must be between P and *n*. Its precise position we are able to determine; for its motion in the small circle, which, seen edgewise, is represented by the straight line PN, is just equal to that of the sun in longitude. Consequently the number of degrees that it has moved from P must be equal to the sun's longitude. The plane of this circle is evidently perpendicular to the surface of the paper, yet for the purpose of bringing the graduation in sight, we will suppose it to turn on the diameter PN, till it lies flat down, as PHN. The reader must not suppose that any circle will be seen in this position by our observer; it is merely drawn so temporarily, for the purposes of measurement. The sun's longitude being 65°.2101, the north pole must be that number of degrees from P, which would bring it to T; and this point, when the circle is turned up again into its place, edgewise upon the paper, would be seen at C, which is hence the position that the north pole must occupy on the disc; and the line CE must represent the northern half of the earth's axis. We shall have no further use for the temporary circle PHN, as drawn, and may, if we

choose, erase it, since it forms no part of the representation that we wished to draw. It merely served to enable us to find the position of the north pole of the earth, which is now effected.

103. The delineation of a parallel of latitude on the disc will show how the diurnal path of a place would appear to our observer. We will select for the purpose that of the Astronomical Observatory of Williams' College, lat 42° 42′ 51″, lon. 73° 12′ 33″ west from Greenwich; the former of which, converted into degrees and decimals by the aid of tables 30 and 31, is 42°.7142. If the latitude were just equal to the sun's declination, the sun would be vertical at noon, and the Observatory would be seen precisely in the centre of the disc at E; but since it exceeds it by 21°.5271, the Observatory must be seen that distance north of the point where the sun is vertical, which when projected on the disc, would become the sine of the arc, measured from E, on the axis EC. To find the length of the sine, we may count the degrees upward from A, and draw the sine DD, the length of which, when applied from E toward C, will reach to the point 12. This point must therefore be the apparent position of the Observatory at noon.

104. If the earth were transparent, it would be seen at midnight considerably farther north, as is evident from an inspection of the globe. The point antipodal to that at which the sun is vertical, and which also would be seen at E, is as many degrees south of the equator as the sun's declination is north. Hence the distance of the Observatory from this point at midnight, must be equal to the latitude of the former added to the sun's declination, which amounts to 63°.9013. This arc, like the former, when projected on the disc, will be seen on the axis EC, equal only to the length of its sine, which we may find in the same way, by counting the number of degrees upward from A, drawing the sine, LL, and laying it off on the axis from E to K. The point K will then represent the apparent place of the Observatory at midnight.

105. The line K-12 will be the shortest diameter of the ellipse, into which the parallel of latitude appears to be thrown by being seen obliquely; the point O, midway between K and 12, its centre, and the line 6O6, drawn through O at right angles to EC, its long-

est diameter. The lines O6, not being foreshortened by being seen obliquely, will appear of the full length of the radius of the parallel, which, we know, is the cosine of the latitude. The complement of the latitude of the Observatory is 47°.2858, and we may find RR, its sine, in the same way as we did the others. Setting off the distance RR each way from O to 6 and 6, we find the extremities of the longest diameter, which must be the points on the disc where the Observatory will be seen at 6 o'clock in the morning, and at the same hour in the evening.

106. Its position at any other hour in the day may be found by the following process. Draw two circles, 6M6 and F′KV′, one on the longest and the other on the shortest diameter of the ellipse, and divide each into 24 parts, in the points 7, 8, 9, 10, &c., corresponding to the hours of the day. Through the division points of the former circle, draw straight lines parallel to EC, the earth's axis; and through those of the latter, at right angles to it. Note the points where the lines that pass through the same hour on both the circles intersect each other, and through them draw the elliptical curve seen in the figure. This curve will represent the parallel of latitude that we wished to delineate, or the path of the Observatory over the disc.* The several points of intersection mark its position at the different hours. The two last circles, with the lines connected with them, except the path of the place, may be drawn in pencil mark, that they may be erased after the latter is drawn, since they are of no further use.

107. The construction we have just completed will show us, if we wish, the time of sunrise or sunset, by noticing at what hour the path of the place cuts the circle of the disc. In this case, it is a little before 5 o'clock in the morning, and a little after 7 in the evening.

108. The moon's latitude (95) is 0°.3664 north. We will therefore take this distance from the scale SS, and measure it from E toward G, which gives X as the place where the centre of the moon

* The reader will readily see that if the sun's declination had been as far south as it is north in this case, the points K and 12 must exchange places, and the curve representing the path of the Observatory must lie on the upper side of 6O6.

will appear to our observer to cross the line EG. The propriety of this step will appear from the following considerations :—

1st. The longitude of the moon, when new, is the same as that of the sun viewed from the earth, or of the earth viewed from the sun; it must therefore be seen in the line EG.

2d. Its latitude is north, therefore it must be seen above the line AB, and not below it, as it would be if its latitude were south.

3d. Its latitude, as given in article 95, is supposed to be measured at the same distance as the other angles for which our scale was made, so that the scale furnishes the proper measure.

The only error that is to be noticed, is that the moon's centre, as seen by our observer projected on the earth's disc, would be a little farther from the ecliptic than its real distance, owing to the divergence of the visual line between the moon and earth. Since however the distance of the sun is so great, the lines drawn from the observer's eye to the centres of the moon and earth must be very nearly parallel, and the divergence just named so small, that it may be disregarded.

109. At the time of our eclipse, the moon's path makes an angle of 5° 44′ 33″ with the ecliptic, tending north. If, therefore, we draw Ey, making an angle of that size with EB, and YZ parallel to it, the latter line will represent the apparent track of the moon's centre across the earth's disc. It passes X at 48 minutes and 44 seconds after 8 in the evening, by Greenwich time, (95,) which, by Williamstown time, is 4 minutes and 6 seconds before 4 in the afternoon. The precise point where it will be at 4 o'clock, or any other instant we may choose to name, may be found from its relative hourly motion, viz., 0°.4649. Taking this distance from our scale, SS, dividing it into 12 equal parts, and thus making the smaller scale, ss, we have its motion in 5 minutes. With the aid of this we can judge by the eye how far it would move in 4 minutes and 6 seconds, and setting off this distance from X toward Z, we find its position at 4 o'clock. Its position at the hours 2, 3, 5, 6, &c., may now be found by measuring off its hourly motion each way from 4. The hourly divisions may now be subdivided at pleasure. In the plate they are divided into quarter hours.

110. The appearance of the moon as projected upon the earth's disc, may be shown by taking its semidiameter, 0°.2479, from the

scale SS, and with it describing a circle, as d b c; selecting for a centre, the point where the moon's centre will be at the time for which we wish to represent its appearance. The plate shows how it will appear at 32 minutes past 5, the time at which its centre crosses the diurnal path of the Observatory, according to our drawing. At that time the Observatory will be at W, a little more than half way from 5 to 6 on its diurnal path, and very nearly coinciding with the moon's centre. It must therefore be invisible to our observer, being hid behind the moon; and the same must be true of a large tract of country about it; for although part of the moon has passed off from the earth's disc, the remaining part covers the parallel of latitude between the hours 3 and 7, and somewhat more. This will amount to over 60° of longitude, reckoning 15° for each hour, which would extend from the Rocky Mountains on the west, about one-third of the way across the Atlantic on the east. Of course the inhabitants of this entire tract must be unable to see the centre of the sun at the time of which we speak, and at Williams' College almost its whole disc must be hid; for it has just been shown that the locality is almost in perfect range with the centres of the sun and moon.

111. It is not difficult to determine precisely what part of the disc will be concealed from view, or eclipsed. Suppose a line to be drawn from the Observatory to the centre of the sun, and a point to be taken in it at the distance of the moon. Let another line be drawn through this point, from one edge of the sun, and continued so as to meet the earth's disc, at a considerable distance, evidently, from the Observatory. If now the upper end of this line be carried round the circumference of the sun, the lower end would describe a circle on the earth's disc, having the Observatory for its centre. And the line itself would describe two similar cones, having a common vertex near the moon, and their bases, one upon the sun, and the other upon the abovementioned circle on the earth's disc. As seen from the centre of the sun, this circle would have precisely the same situation in respect to the moon, that the sun would have as seen from the Observatory; so that if a portion of this circle be hid from our observer at the sun, by some intervening object, a like portion of the sun would be hid when viewed from the Observatory. We wish then, to determine the size and position of this circle at 32 minutes past 5, that we may see how large

a part of it is covered or concealed by the moon. As viewed from the fixed point spoken of, near the moon, it and the sun would evidently subtend the same angle. But the sun would subtend very nearly the same angle, whether seen from that point or from the earth, for the relative distances are nearly the same. Now the sun's semidiameter, seen from the earth, at that time subtends an angle of 0°.2635; therefore this circle must, at the distance of the moon, subtend the same angle. Hence we may take this distance from the scale SS, and with it discribe the circle a b c, from the centre W, the position of the Observatory at the time, and we have the circle in question; showing that a very slender crescent of light will be seen on the south side of the moon.

The width of this crescent is usually described, by dividing the diameter of the circle into 12 equal parts, called digits, and seeing how many of these it contains. In this case it contains about one-fourth of one of these divisions, leaving 11 3-4 digits eclipsed.

112. The eclipse must evidently commence at Williams' College, as soon as the moon and the circle that we last drew begin to interfere, which must be as soon as the distance between their centres becomes less than the sum of their semidiameters. The two semidiameters added together make 0°.5114, and we may take this distance from the scale SS, and setting one foot of our compasses on the moon's path, some distance to the left of W, and the other on that of the Observatory, move them backward or forward till both feet stand on the same hour and minute, which must be the time when the eclipse commences. By a similar operation at the right hand of W, the time of the end may be found.

The results, according to our drawing, are as follows:—

	h. m.
Beginning, - - - - - - - -	4 15
Greatest obscuration, - - - - -	5 32
End, - - - - - - - - -	6 38
Duration, - - - - - - - -	2 23
Digits eclipsed, - - - - - - -	11 3-4

113. We may derive from Fig. 18th a method, by which the size and appearance of a solar eclipse, at any given time, and place may be calculated mathematically. The tabular latitudes and longitudes of the sun and moon are calculated for the centre of the

earth, and are consequently correct only where the sun and moon are vertical, or in the zenith. At all other places they would be affected by parallax. If the place were situated so as to appear on the disc above or below AB, the parallax would affect the latitude, and if on the right or left of GE, the longitude would be affected. Thus V is the place of the Observatory at 3 o'clock P. M. on the day of our eclipse, and E that at which the sun is vertical and the moon nearly so, not differing from it more than about $\frac{1}{2}°$. Hence VE is the sine of the zenith distance, to which the total effect of parallax is always proportional. Draw VX at right angles to AB, and it will represent the effect of parallax upon the latitude of the sun and moon, and EX upon the longitude.

Having computed the elements (95) for the time at which we wish to represent the appearance of the eclipse, the arc GP, which is equal to the obliquity of the ecliptic to the equator, will be known, and calling the radius of the semicircle AGB unity, P*n* and E*n* can be found. Again, since P*n*, the radius of the circle NHP, is now known, and also the arc PT, being equal to the sun's longitude, C*n* can be found. Then in the right angled triangle E*n*C, the two sides E*n* and C*n* are known, and we can find the angle GEC.

If we let *s* represent the sun's longitude, and *m* the obliquity of the ecliptic to the equator, then tan. GEC=tan. $m \times$cos. *s*.

The lines EK and E12 are the sines of the sum and difference of the latitude of the place and the sun's declination, and are therefore known: hence EO, which is equal to half the sum, can be found.

The line O6 is known, being the cosine of the latitude, and also the arc MF, being the hour arc from noon, or the interval between noon and the time to which the calculations refer converted into degrees; hence OI, which is the sine of this arc, or its equal UV, and also IF its cosine may be found. IV is equal to IF foreshortened in the ratio of radius to the sine of the sun's declination, and is therefore known; and subtracting* it or its equal OU from EO, we shall have EU. Now in the right angled triangle EUV, the sides EU and UV are known, and we can find the side EV and the angle UEV.

* If the sun's declination and the latitude of the place are both north or both south, subtract; but if one is north and the other south, add.

If we let l represent the latitude of the place, d the sun's declination, and t the hour arc from noon, then

$$UV=\sin. t\times\cos. l$$

$$EU=\sin. l\times\cos. d\pm{}^{*}\cos. l\times\sin. d \cos. t$$

$$\tan. UEV=\frac{\sin. t}{\text{cotan}. l\times\cos. d\pm{}^{*}\sin. d\times\cos. t}$$

We now have the angles GEC and UEV, which, added together and subtracted from 90°, leave the angle VEX. We have also the line EV. Hence, in the right angled triangle VEX, we are enabled to find EX and VX. Since the parallax of a heavenly body at any altitude is equal to the horizontal parallax mutiplied by the sine of the zenith distance,† which in this case is EV, it follows that if we multiply it by EX, we shall get the effect of parallax on the longitude, and if by VX, on the latitude.

The value of EV and the angle UEV may be obtained, if preferred, by another process. The co-latitude of the place of observation, the co-declination of a heavenly body and its zenith distance form a spherical triangle, in which the two former parts are in this case known; and also the included angle, being the hour angle from noon. Hence the third side can be found, the sine of which is EV, and the remaining angles. Now, by the principles on which Fig. 18 is constructed, the angle UEV is the same as that opposite the co-latitude in the sperical triangle, and is therefore known.

Having corrected the latitudes and longitudes of the sun and moon for the effect of parallax, their differences will form two sides of a right angled triangle, and the distance between their apparent centres will be the hypothenuse. By comparing the latter with the apparent semidiameters of the sun and moon, the size of the eclipse can be readily determined.

Our 8th element is the moon's apparent semidiameter, as seen from the centre of the earth; but the distance of the moon from any place on the earth's surface at which it is visible (save when it is in the horizon) is less than from the centre, which must cause it to subtend a greater angle. The augmentation is a maximum when the moon is in the zenith, and grows less when it recedes from it; hence the sine of the zenith distance, EV, is the proper

* See note on preceding page.

† Olmsted's Astronomy, Art. 82.—Herschell's do., Art. 303.—Norton's do., Art. 98.—Gunmeare's do., Chap. v., Art. 5.

argument for determining the amount of the augmentation, and is so used in table 29th.

We will show the results of such calculations as we have been describing, by applying them to our eclipse at 32 minutes past 5, the time at which it is represented in Fig. 18th. After computing the necessary elements for the time and the parallax, as above described, we have the following :—

	Sun.	Moon.		Sun.	Moon.
Longitude,.........	65°.2744	66°.0189	Latitude,..........	0°.0000	0°.4369 north
Parallax,...........	.0020	.7644	Parallax,..........	.0011	.4154
	65°.2724	65°.2545		0°.0011 south	0°.0215 north
	65°.2545			0°.0215 north	
Difference,.........	0°.0179		Difference,.........	0°.0226	

Making these differences the legs of a right angled triangle, and regarding them as straight lines, which we very safely do, since they are so small, we find the hypothenuse to be 0°.0288, which is the apparent distance between the centres of the sun and moon.

The sine of the zenith distance is found by the calculations to be .95678, and the consequent augmentation of the moon's semidiameter, taken from table 29th, 0°.0007. The moon's apparent semidiameter would otherwise be at this time 0°.2478, and applying this correction, it becomes 0°.2485. The sun's remains the same as was found in article 95th, viz., 0°.2635, and the difference between the two semidiameters is 0°.0150. By comparing this difference with the distance between the centres, viz., 0°.0288, we see that the sun's disc must extend beyond the moon's on one side by 0°.0438, or about 2′ 38″, while on the other it would fall short by 0°.0138; thus forming a slender crescent of light about the moon, as we have before remarked.

CHAPTER XIII.

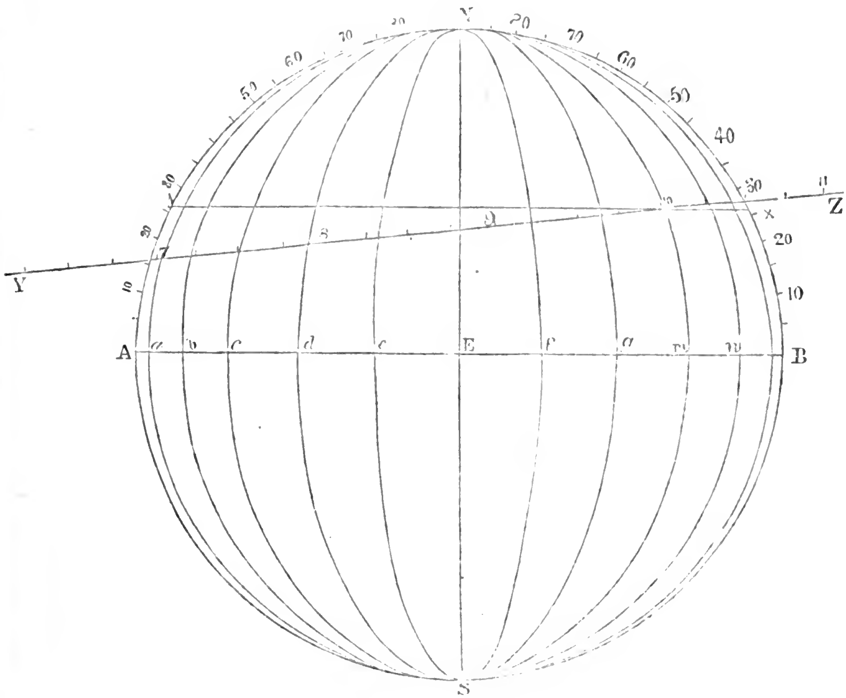
CENTRAL TRACK OF A SOLAR ECLIPSE.

114. If we examine the plate principally referred to in the last chapter, (Fig. 18,) we notice that the moon's track crosses the path of the Astronomical Observatory of Williams' College twice; once about 20 minutes before 4, and again about 32 minutes past 5. In the first instance, it crosses far west of where the Observatory will be at the time, and in the second, a little east. Counting every hour as 15° of longitude, the point where it first crosses is about 70° west of the Observatory, which carries it into the Pacific ocean, not far from Astoria. It crosses the second time at a point about 3° east of the Observatory, which is in the Atlantic ocean, off Cape Ann. If we had numerous elliptical curves drawn to represent the different parallels of latitude, we might, by a process analagous to the foregoing, determine over what countries of the earth the moon's centre, or more strictly, the centre of its shadow would pass, from the time it first struck the disc on the west side, till it passed off on the east. There is however an easier way of effecting this, by means of a figure of different construction, used in connection with a terrestrial globe.

115. Let ANB (Fig. 19) represent the northern half of the earth's disc, as seen from the sun, YZ the moon's track, with the hours of Greenwich time marked on it, AB a portion of the ecliptic, as in Fig. 18, in the last chapter, and the curved lines secondaries to it, orthographically projected at intervals of 15°.

Now to adjust the globe so as to correspond with this figure, elevate the north pole, as directed in the last chapter, (99,) and at the point that answers to N in the figure, 90° above the wooden horizon, which now represents the ecliptic, or 23½° from the north pole of the earth, screw on the graduated quadrant of altitude. By swinging the other end round, between the globe and the inside of the wooden horizon, it may be made to represent any of the lines NA, N*a*, N*b*, &c. Or we may screw it on at S, and thus represent the other half of the curves, which becomes necessary when the moon's latitude is south.

Fig. 19.



The point E in the figure corresponds, in the present case, to that marked May 26th on the wooden horizon, and consequently the points A and B must be found 90° from it on each side. If the lower end of the quadrant of altitude is brought to one of these points, we shall have a representation of the arc NA ; and if to the other, of NB. Our figure shows that the moon's track crosses the former about 16° 40′ above A, and the latter about 29° above B. The graduation of the quadrant of altitude would readily show where these places are on the globe, were it not for its diurnal revolution, which we must next take into account.

116. It is evident that the sun must always be just rising at all places situated on the line NA, and just setting at all on the line

NB. Now, at the equator, the sun rises invariably at 6 o'clock in the morning, and sets at the same hour in the evening. Therefore, at 8 minutes before 7 in the evening, by Greenwich time, when, according to our drawing, the moon's centre first strikes the earth's disc, it must be just 6 o'clock in the morning at the place where the equator cuts the arc NA, (compare Fig. 17.) The question is then reduced to this, viz., at what place on the equator is it 6 o'clock in the morning, at the same time that it is 8 minutes before 7 in the evening by Greenwich time? Converting the time into longitude, the point in question is found to be in the Pacific ocean, a little southwest of Mulgrave's island, in longitude 167° east from Greenwich.

Turn the globe on its axis till this point is brought under the quadrant of altitude, (the latter being adjusted so as to represent NA.) and count 16° 40′ upward from the wooden horizon, and we shall discover the place where the centre of the eclipse first strikes the earth. The experiment shows it to be near the Caroline Islands, lat. 7° north, and lon. 164° east.

117. According to our drawing, the centre of the eclipse leaves the earth at 29 minutes past 10 in the evening, by Greenwich time, but at the point where the equator cuts the arc NB it is but 6 o'clock in the evening. The longitude corresponding to this difference in time is $67\frac{1}{4}$° west, which is in the southern part of Venezuela, in South America. Now turning the globe on its axis till this point is brought under the graduated quadrant, (adjusted so as to represent NB,) and counting upward 29° from the wooden horizon, we find that the eclipse leaves the earth in the Atlantic ocean, about 800 miles easterly from Bermuda.

118. Let it now be required to find where the eclipse is central at any time during its passage across the disc; suppose at 10 o'clock P. M.

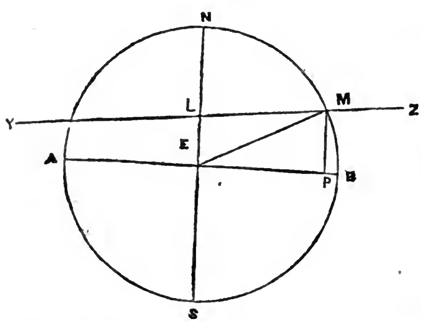
Through the point of division representing 10 o'clock on the moon's track, draw sx parallel to AB. The arc Bx contains about $27\frac{1}{2}$°, which must also be the number of degrees in the arc m 10. Bring the foot of the quadrant of altitude to m, which is 45° from E. Turn the globe backward on its axis $7\frac{1}{4}$° from its last position, such being the amount of its motion between 10 o'clock and 29 minutes past 10, the time for which we last calculated. The

$27\frac{1}{2}$th degree on the quadrant of altitude, reckoned upward from the wooden horizon, will mark where the eclipse must be central at 10 o'clock. We find, on trial, that it is near the west end of Lake Superior, in North America.

119. These mechanical methods give a tolerable approximation to the point where a solar eclipse will be central at any time; and by taking a sufficient number of points, we may delineate its general track. But where much accuracy is required, recourse must be had to calculation. The theory of the following process is the same as that of the mechanical method just employed.

120. The first step is to find the time when the centre of the moon's shadow first strikes or leaves the earth,* (I will here adopt the latter,) and the sun's longitude at the time. The calculation is a very easy one, if the moon's track across the disc is considered as a straight line, and the earth as a perfect sphere.

Fig. 20.



Let ASBN (Fig. 20) represent the earth's disc, and YZ the moon's track across it. In the triangle ELM, the two sides, EL and EM are known, the former being the moon's latitude at the time of new moon, and the latter its equatorial parallax. Also, the angle ELM is known, being equal to that of the moon's path with the ecliptic, increased by 90°. Hence we can find the side LM, and the angle LEM, or its complement MEB. Knowing the moon's hourly motion, we can easily tell how long it would take it to pass from L to M, which, added to the time of new moon, will give the time of its leaving the earth.

The sun's motion in longitude during this interval can be calculated from its hourly motion, and thus its longitude at the time the eclipse leaves the earth will be known.

If still greater accuracy be required, the moon's latitude, and

* It is immaterial whether we take the time when the centre of the shadow strikes or leaves the earth, provided we make the other parts of the process to correspond.

the difference in the longitudes of the sun and moon, at the time last found, can be computed from the tables; the former of which is represented by the line MP, and the latter by EP. Also we may compute again the angle of the moon's path with the ecliptic, and the horary motions. With these data we could easily calculate the distance of the point M from the circumference, reckoned on the line YZ, and how long it would take the moon to pass over it.

121. Next, find the longitude of the sun, and the latitude and longitude of the moon at the time for which we wish to calculate the position of the centre of the eclipse. This may be done either from their hourly motions and the angle of the moon's path with the ecliptic, or, if greater accuracy be required, directly from the tables.

Let the sun's longitude thus found $= s$.

Let the moon's latitude do. $= l$.

Let the difference in the longitudes of the sun and moon $= d$.

Let the sun's longitude at the time the eclipse leaves the earth $= s'$.

Convert the time to which the calculations refer into degrees, minutes and seconds, reckoning 15° for an hour; subtract therefrom 90°, (borrowing 360° if necessary,) and let the remainder $= t$.

Let the moon's equatorial parallax, which may be regarded as constant during the eclipse, $= p$.

Let the obliquity of the ecliptic to the equator $= m$.

Let P, P_1, P_2, &c. = sundry arcs and angles obtained during the process of computation.

Let the required latitude of the centre of the shadow $= x$.

Let the required longitude, reckoned westerly from the meridian of Greenwich, or of that place for which the time is given, $= y$.

Then $\frac{l}{p}=\sin. \mathrm{P}$,

$\frac{d}{p \times \cos. \mathrm{P}}=\sin. \mathrm{P}_1$,

$^{*}s \pm \mathrm{P}_1=\mathrm{P}_2$,

$\sin. m \times \cos. \mathrm{P}_2=\sin. \mathrm{P}_3$,

$\tan. m \times \sin. \mathrm{P}_2=\tan. \mathrm{P}_4$,

$^{\dagger}\sin. (\mathrm{P} \pm \mathrm{P}_4) \times \cos. \mathrm{P}_3=\sin. x=$ the latitude,

$\frac{\cot. s'}{\cos. m}=\tan. \mathrm{P}_5$,

$\frac{\tan. \mathrm{P}_2}{\cos. m}=\mathrm{P}_6$,

$\frac{\sin. \mathrm{P}_3}{\cos. x}=\mathrm{P}_7$,

$\sin. x \times \tan. \mathrm{P}_7=\mathrm{P}_8$,

$^{\ddagger}\mathrm{P}_5-\mathrm{P}_6+t \pm \mathrm{P}_8=y=$ the longitude.

The chief difficulty in applying these equations consists in knowing which of the four possible values to give to P, P_1, P_2, &c. The following statements will remove all doubt.

P, P_1, P_3 and P_4 are each always less than 90°.

P_5 is always of the same affection as s' increased by 90°.

P_6 is always of the same affection as P_2.

P_7 is less or greater than 90°, according as P is less or greater than the complement of P_4; it never exceeds 180°.

P_8 is always of the same affection as P_7.

122. It is impossible, by a mere description, to convey to the reader a clear idea of the reason of the several steps of this process; but if he will take his globe, and adjust it in the same manner as he would do to find the position of the centre of the eclipse by the previous mechanical process, he may be able to discover suc-

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* If after the new moon +; if before it —.

† The sign before P_4 is + if P_2 is less than 180°; but — if it is greater. And in the latter case if P_4 is greater than P, the latitude of the place will be opposite in character to that of the moon; i. e. if the moon's latitude is north that of the place will be south, and the contrary.

‡ The sign before P_8 is + if P_2 is between 0° and 90°, or between 270° and 360°; but — if P_2 is between 90° and 270°.

cessively the arcs and angles expressed by P, P_1, &c.; and hence to understand the method by which they are obtained.

P is the distance of the centre of the eclipse from the ecliptic, measured on a secondary to it drawn upon the earth's surface, as *m* 10, (Fig. 19.) Or, it is the latitude of that point in the heavens, on which an observer, placed at the centre of the earth, would see the centre of the shadow at the earth's surface projected, if the earth were transparent.

P_1 is an arc of the ecliptic, intercepted between the aforesaid secondary and the point where the sun is vertical. Or, it is the difference between the sun's longitude and that of the aforesaid point in the heavens.

P_2 is the same arc increased by the sun's longitude. Or, it is the longitude of the aforesaid point in the heavens.

P_3 is an arc of a great circle, drawn from the north pole of the equator, perpendicular to the aforesaid secondary.

P_4 is the arc of the secondary, intercepted between this perpendicular and the north pole of the ecliptic.

P_5 is the right ascension of that point in the equator where it is cut by a secondary to the ecliptic passing through the centre of the shadow on the earth's surface, at the time that it leaves the earth; or, it is the right ascension of that point in the heavens on which the centre of the shadow would be seen projected at that time, by an observer at the centre of the earth.

P_6 is the right ascension of that point in the equator, where it is cut by the first mentioned secondary.

P_7 is the angle at the centre of the shadow, or at the first mentioned point in the heavens, contained between secondaries to the ecliptic and equator passing through that point.

P_8 is the arc of the equator intercepted between these two secondaries.

133. To apply the process to a particular case, let it be required to find the place where the solar eclipse which we have taken as an example, will be central at 20 minutes and 51 seconds past 10,

by Greenwich time. After the preparatory steps described in articles 120 and 121, we have the following data and results :—

DATA.	RESULTS.
Time=10*h*. 20*m*. 51*sec*.	$P = 28° \; 58' \; 22''$
$s'=65° \; 16' \; 50''$	$P_1 = 66 \; 29 \; 27$
$s=65° \; 16' \; 34''$	$P_2 = 131 \; 46 \; 1$
$l= 0°.4394$	$P_3 = 15 \; 22 \; 40$
$d= 0°.7278$	$P_4 = 17 \; 56 \; 12$
$t=65° \; 12' \; 45''$	$x = 44 \; 45 \; 28$ = the latitude.
$p= 0°.9072$	$P_5 = 153 \; 21 \; 5$
$m=23° \; 27' \; 34''.5$	$P_6 = 129 \; 19 \; 34$
	$P_7 = 21 \; 55 \; 45$
	$P_8 = 15 \; 49 \; 38$
	$y = 73 \; 24 \; 38$ = the longitude.

Thus we find that the centre of the shadow, at the time just mentioned, is in lat. 44° 45′ 28″, and lon. 73° 24′ 38″, which is on the west shore of Lake Champlain, about four miles north of the village of Plattsburg.

124. In bringing to a close our examination of the solar eclipse of 1854, it may not be uninteresting to give a general description of it.

The centre of the eclipse first strikes the earth at 54 minutes and 27 seconds past 6 P. M., (Greenwich time,) in the Pacific ocean, not far from the Caroline Islands, and travelling northeastwardly, nearly over the Sandwich Islands, strikes the American coast a little north of Astoria, in Oregon Territory, about 24 minutes past 9. Crossing the Oregon Territory, it enters the British possessions, and turning easterly, and then southeasterly, re-enters the United States territories west of Lake Superior. At 13 minutes past 10 it crosses the outlet of Lake Superior, about 100 miles N. W. from Michilimackinack. After reaching the settled parts of Canada, it passes a little south of Bytown, travelling at a rate of 70 miles per minute, and reaches the St. Lawrence about 20 miles below St. Regis, at 20 minutes past 10. Again entering the United States, on the north line of New York, it arrives at the west shore of Lake Champlain, about four miles north of Plattsburgh, at 20 minutes and 51 seconds past 10. It strikes the opposite shore in

the south part of the town of Georgia, and from thence passes through the following towns in Vermont, viz., Fairfax, Fletcher, Cambridge, Stirling, Morriston, Elmore, Woodbury, Cabot, Danville, Barnet, Waterford, and reaches the Connecticut river at 21 minutes and 45 seconds past 10. Travelling now about 100 miles per minute, it passes through the towns of Littleton and Bethlehem, in New Hampshire, and from thence directly over the Notch in the White Mountains, and through Adams and Chatham into Maine. After passing through Fryeburg, Denmark, Bridgetown, Sebago, and across the pond into Windham, Gray, Cumberland, and Yarmouth, it strikes the Atlantic in the latter town, about ten miles, in a direct line, from Portland. It leaves the earth at 28 minutes and 55 seconds past 10, in lat. 32° 36′ 6″, lon. 49° 44′ 12″, which is in the Atlantic ocean, about 800 miles east of Bermuda.*

Since the apparent size of the moon at the time of the eclipse is less than that of the sun, (95,) the eclipse cannot be total at any place; but along the line we have described, the visible part of the sun will appear as a very slender bright ring, encircling the moon. This appearance will extend about fifty miles on each side, taking in Burlington, Middlebury, Dartmouth, Bowdoin, and Waterville colleges; the ring will appear of uniform width only along the central line. Such eclipses as this are called *annular*.

CHAPTER XIV.

DELINEATION OF A LUNAR ECLIPSE.

125. The delineation of a lunar eclipse is extremely simple, since it consists merely in representing the passage of the moon across the earth's shadow. To show the method of effecting it, we will proceed to delineate the lunar eclipse of November, 1844, from the elements obtained in chapter 11th. The angular dimen-

* By taking into account several minute circumstances that we have disregarded, the central track of the eclipse may vary slightly from this description, probably passing ten or fifteen miles further north, and nearly over Bowdoin college.

sions given in the 4th, 6th, 8th and 10th elements being supposed to be all taken at the same distance, viz., the distance from the earth to the moon, will serve as a measure for their absolute dimensions, in the same manner as they did in the solar eclipse.

The first step is to make the larger and smaller scales SS and *ss* (Fig. 21) just as was done in the solar eclipse, (101 and 109.) Take from the longer one the semidiameter of the earth's shadow, viz., 0°.6306, and with it describe the graduated circle BDAE, which will represent the shadow. It is evident that the plane of the ecliptic must bisect the shadow, and we therefore draw the two diameters AB and DE at right angles to each other, the former to represent a section of the plane of the ecliptic, and the latter its axis.

126. The moon's latitude is 0°.1810 north. We therefore take this distance from the scale SS, and measure it upward from C toward D, which gives M as the place of the moon's centre at the time of full moon. If the latitude were south, the centre would be found in the line CE.

127. Its path makes an angle of 5° 45′ 41″ with the ecliptic, tending south. If, therefore, we draw CF, making an angle of that size with CB, and YMZ parallel to it, the latter line will represent the track of the moon's centre across the shadow. It passes M at 40 minutes and 14 seconds past 11 in the evening, by Greenwich time, and its position at any other hour and minute may be found by graduating the line YZ, as directed in article 109th.

128. By taking the moon's semidiameter, 0°.2453, from the scale SS, and with it describing a circle from any point in its path as a centre, the position of the entire disc will be shown, as it must exist at the time indicated at its centre on the path. It is drawn in the plate in five different positions: 1st, when it begins to impinge on the shadow at G, which must be the commencement of the eclipse: 2d, when it just falls wholly within the shadow at H, at which time the eclipse must begin to be total: 3d, when its centre is at N, found by drawing CN perpendicular to YZ, and thus (Euc. 3, 2) bisecting the chord, which must be the middle of the eclipse: 4th, when it begins to leave the shadow at L, at which time it

must cease to be total, and 5th, when it entirely leaves the shadow at P, which must be the end of the eclipse. The respective centres are at R, S, N, T and V, and the time may be determined very nearly by the drawing.

129. More accurate results may, however, be obtained by calculating the length of MR, MS, MN, MT and MV, and then finding by the relative hourly motion of the moon, how long it must take it to pass over them. In the right angled triangle CNM, the side CM and the angle MCN are known, being our 4th and 11th elements, and we can find CN and MN. The side CN is common to the two right angled triangles SNC and RNC, and the sides CS and CR are also known, the former being the difference, and the latter the sum of our 8th and 10th elements. Hence NS and NR can be found, and likewise their equals NT and NV. Now, by adding and subtracting MN, which is known, we shall have the lines required.

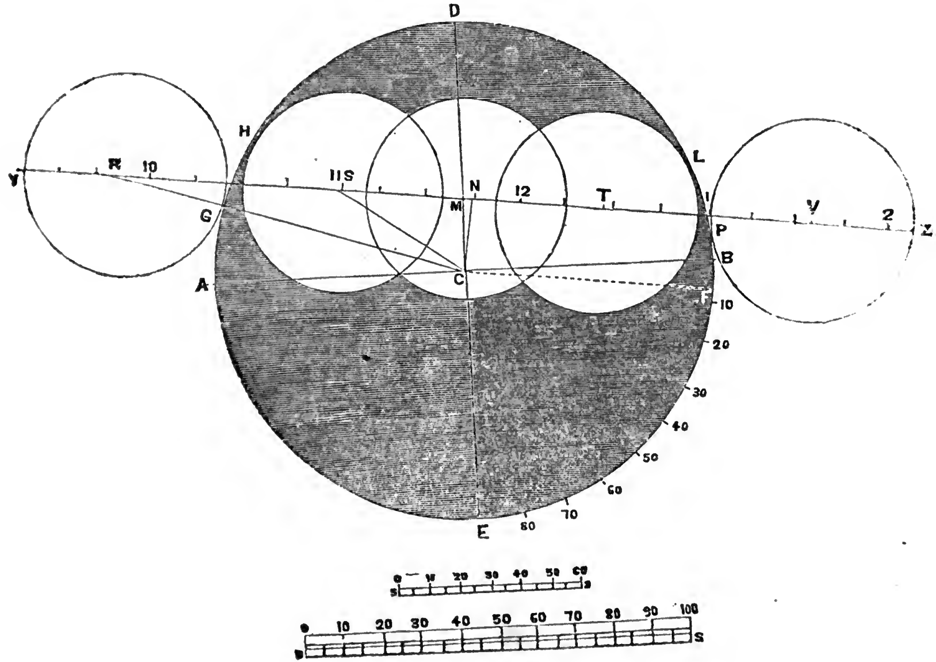
130. The times obtained by this process are as follows :—

	h.	m.	sec.
Commencement of the eclipse, - -	9	48	59
Begins to be total, - - - - - -	10	57	31
Middle, - - - - - - - -	11	42	42
Ceases to be total, - - - - - -	12	27	53
End of the eclipse, - - - - - -	1	36	24
Duration of total obscuration, - - - -	1	30	22
Duration of the eclipse, - - - - -	3	47	26

The foregoing is Greenwich time, but can readily be converted into that of any other place, by allowing for the difference of longitude.

If strict accuracy were aimed at, the elements should be calculated at several intervals during the eclipse, as they are liable to vary considerably.

Fig. 21.—LUNAR ECLIPSE, NOVEMBER, 1844.



EXPLANATION OF TERMS,

AS USED IN THIS WORK.

Ecliptic. The apparent annual path of the sun through the heavens.

Nodes. The points where the orbit of a planet intersects the plane of the ecliptic.

Ascending Node. That through which the planet passes from the south side of the ecliptic to the north side.

Descending Node. That through which it returns to the south side.

Line of the Nodes. A straight line connecting the two nodes.

Equinoctial Points, or Equinoxes. The point where the ecliptic intersects the plane of the equator.

Vernal Equinox. That through which the sun apparently passes from the south side of the equator to the north side.

Autumnal Equinox. That through which it returns to the south side.

Solstitial Points, or Solstices. Points in the ecliptic midway between the equinoxes.

Perigee. The point where the sun or moon approaches nearest the earth.

Apogee. The point where they are most distant from the earth.

Apsis. The common name for apogee or perigee.

Apsides. The plural of apsis.

Line of the Apsides. A straight line joining the two apsides.

Conjunction. In the same direction as the sun.

Opposition. In an opposite direction from the sun.

Syzygy. The common name for conjunction and opposition.

Quadrature. Points in the moon's orbit midway between the syzygies.

Radius Vector. A straight line drawn from a revolving body to the centre about which it revolves.

Latitude of a heavenly body. Its distance north or south of the ecliptic.

Longitude of a heavenly body. Its distance eastwardly from the vernal equinox, measured on the ecliptic.

Right Ascension. The same, measured on the equator.

Declination. The distance of a heavenly body, north or south, from the equator.

Lunation. The time from one new or full moon to another.

Parallax. The apparent change in the place of a heavenly body, when viewed from different points. It is always equal to the angle which a line connecting the points of observation would subtend, when viewed from the body.

Note.—It is thought best to omit, in the present edition of this work, a sequel, or second part, now in manuscript, and to which there have been several references in the foregoing pages, explaining the method of calculating most of the lunar motions and inequalities directly from the laws of elliptical motion and the principles of gravitation, without the aid of tables. It may appear hereafter.

Errata.—Page 15th, &c. for elipse read ellipse; page 27, near the bottom, for syzygyes read syzygies.

Page 32, Fig. 11, the lines NE and EP should be in the same straight line.

ASTRONOMICAL TABLES.

LIST OF TABLES.

TABLE I.

Elements of Orbits of Sun and Moon.

	Sun.	Moon.
	° ′ ″	° ′ ″
Mean longitude, Jan. 1, 1801, - - - - -	280 39 13.17	118 17 8.3
Motion in 100 years or 36525 days, - - - - -	36000 46 0.77	481267 52 41.6
Mean longitude of perigee, Jan. 1, 1801, - -	279 31 9.71	266 10 7.5
Motion of do. eastward in 100 years, - - - -	1 42 56.0	4069 2 46.6
Longitude of Moon's Node, Jan. 1, 1801, - -		13 53 17.7
Motion of do. westward, in 100 years, - - - -		1934 9 57.5

TABLE II.

Mean New Moon, &c. in March.

Year.	Mean New Moon in March.				Sun's Mean Anomaly.	Moon's Mean Anomaly.	Sun and Moon's Mean Longitude.	Longitude of Node.
	d.	*h.*	*m.*	*sec.*	°	°	°	°
1800	25	0	18	49	83.208	127.960	2.7139	28.8207
1810	5	6	36	43	63.167	63.441	342.8436	196.4758
1820	14	1	38	40	72.232	24.740	352.0799	2.5671
1830	23	20	40	37	81.295	346.038	1.3163	168.6585
1840	3	2	58	31	61.254	281.520	341.4458	336.3135
1841	22	0	31	8	79.624	257.140	359.8335	315.9844
1842	11	9	19	42	68.888	206.943	349.1144	297.2190
1843	0	18	8	17	58.152	156.746	338.3954	278.4536
1844	18	15	40	54	76.522	132.366	356.7831	258.1245
1845	8	0	29	28	65.786	82.169	346.0640	239.3591
1846	26	22	2	6	84.156	57.789	4.4517	219.0300
1847	16	6	50	40	73.420	7.592	353.7326	200.2646
1848	4	15	39	15	62.684	317.395	343.0135	181.4991
1849	23	13	11	52	81.054	293.015	1.4012	161.1702
1850	12	22	0	27	70 318	242.819	350.6822	142.4048
1851	2	6	49	1	59.583	192.622	339.9631	123.6394
1852	20	4	21	38	77.952	168.242	358.3508	103.3103
1853	9	13	10	13	67.217	118.045	347.6317	84.5449
1854	28	10	42	50	85.586	93.665	6.0194	64.2158
1855	17	19	31	24	74.851	43.468	355.3003	45.4504
1856	7	4	19	59	64.115	353.271	344.5813	26.6851
1857	25	1	52	36	82.485	328.891	2.9689	6.3560
1858	14	10	41	11	71.749	278.694	352.2499	347.5906
1859	3	19	29	46	61.013	228.497	341.5308	328.8252
1860	21	17	2	24	79.383	204.117	359.9185	308.4961
1861	11	1	50	58	68.647	153.920	349.1994	289.7307
1862	0	10	39	33	57.911	103 723	338.4803	270.9653
1863	19	8	12	10	76.281	79.343	356.8680	250.6362
1864	7	17	0	45	65.545	29.146	346.1490	231.8708
1865	26	14	33	22	83.915	4.766	4.5366	211.5417
1866	15	23	21	57	73.179	314.569	353.8176	192.7763
1867	5	8	10	31	62.443	264.372	343.0985	174.0110
1868	23	5	43	9	80.813	239.992	1.4862	153.6819
1869	12	14	31	43	70.077	189.795	350.7671	134.9165
1870	1	23	20	18	59.341	139.599	340.0481	116.1511
1871	20	20	52	55	77.711	115.219	358.4357	95.8220
1872	9	5	41	29	66.976	65.022	347.7167	77.0567
1873	28	3	14	6	85.346	40.642	6.1043	56.7275
1874	17	12	2	41	74.610	350.445	355.3853	37.9622
1875	6	20	51	15	63.874	300.248	344.6662	19.1968
1876	24	18	23	53	82.244	275.868	3.0539	358.8677
1877	14	3	12	28	71.508	225.671	352.3348	340.1023
1878	3	12	1	3	60.772	175.474	341.6158	321.3370
1879	22	9	33	40	79.142	151.094	0.0034	301.0079
1880	10	18	22	15	68.406	100.897	349.2844	282.2425
1881	0	3	10	50	57.670	50.700	338.5653	263.4771
1882	19	0	43	27	76.040	26.320	356.9530	243.1480
1883	8	9	32	2	65.304	336.123	346.2339	224.3826
1884	26	7	4	39	83.674	311.743	4.6216	204.0535
1885	15	15	53	14	72.938	261.546	353.9025	185.2881
1886	5	0	41	49	62.202	211.349	343.1835	166.5228
1887	23	22	14	26	80.572	186.969	1.5712	146.1937
1888	12	7	3	0	69.836	136.772	350.8521	127.4283
1889	1	15	51	35	59.100	86.575	340.1330	108.6629
1890	20	13	24	12	77.470	62.196	358.5207	88.3337
1891	9	22	12	47	66.734	11.999	347.8016	69.5683
1892	27	19	45	24	85.104	347.619	6.1893	49.2392
1893	17	4	33	59	74.368	297.422	355.4703	30.4739
1894	6	13	22	33	63.632	247.225	344.7512	11.7085
1895	25	10	55	10	82.002	222.845	3.1339	351.3794
1896	13	19	43	45	71.266	172.648	352.4198	332.6140
1897	3	4	32	19	60.530	122.451	341.7007	313.8486
1898	22	2	4	56	78.900	98.071	0.0884	293.5195
1899	11	10	53	31	68.164	47.874	349.3694	274.7542
1900	0	19	42	6	57.428	357.677	338.6503	255.9888

This Table shows the time of New Moon in March, of each year, with the longitudes, anomalies, &c. of the Sun and Moon at that time, on the supposition that all the motions are performed with uniform angular velocity.

TABLE III.

Mean Lunations.

No. Lun.	Mean Lunations. d.	h.	m.	sec.	Sun's Mean Anomaly.	Moon's Mean Anomaly.	Sun and Moon's Mean Longitude.	Longitude of Node.
1	29	12	44	3	29°.105	25°.817	29°.1067	1°.5638
2	59	1	28	6	58.211	51.634	58 2135	3.1275
3	88	14	12	9	87.316	77.451	87.3202	4.6914
4	118	2	56	12	116.421	103.268	116.4270	6.2551
5	147	15	40	14	145.527	129.085	145.5337	7.8189
6	177	4	24	17	174.632	154.906	174.6405	9.3827
7	206	17	8	20	203.738	180.718	203.7472	10 9465
8	236	5	52	23	232.843	206.535	232.8530	12.5102
9	265	18	36	26	261.948	232.352	261.9607	14.0740
10	295	7	20	29	291.054	258.169	291.0674	15.6378
11	324	20	4	32	320.159	283.986	320.1742	17.2016
12	354	8	48	35	349.264	309.803	349.2809	18.7654
13	383	21	32	37	378.370	335.620	378.3877	20.3291
½	14	18	22	1	14.553	192.908	*14.5534	.7819

NOTE.—The true quantities for one lunation, from which these tables are calculated, are as follows, viz.

Length of a lunation, - - -		29*d.* 12*h.* 44*m.* 2.88*sec.*
Sun's mean motion in Anomaly in one lunation,	-	29°.10535764
Moon's do. - - - - -	-	25 .81692410
Sun and Moon's mean motion in Longitude, do.	-	29 .10674457
Mean motion of the Node, do. - -	-	1 .56377989

TABLE IV.

Mean Motions of the Sun and Moon.

Days.	Sun's Anomaly.	Sun's Longitude.	Moon's Anomaly.	Moon's Longitude.	Longitude of Node.
1	0°.9856	0°.98565	13°.0650	13°.17640	.0°5295
2	1.9712	1.97129	26.1300	26 35279	.10591
3	2.9568	2.95694	39.1950	39.52919	.15886
4	3.9424	3.94259	52 2600	52 70559	.21182
5	4.9280	4.92824	65.3250	65.88198	.26477
6	5.9136	5.91388	78.3900	79.05838	.31773
7	6 8992	6.89953	91.4549	92.23477	.37068
8	7.8848	7.88518	104.5199	105.41117	.42364
9	8 8704	8.87083	117.5849	118.58757	.47659

Hours.	Sun's Anomaly.	Sun's Longitude.	Moon's Anomaly.	Moon's Longitude.	Longitude of Node.
1	.0°411	.0°4107	0°.5444	.5°4902	.0°0221
2	.0821	.08214	1.0887	1.09803	.00441
3	.1232	.12321	1.6331	1.64705	.00662
4	.1643	.16427	2.1775	2.19607	.00883
5	.2053	.20534	2.7219	2.74508	.01104
6	.2464	.24641	3.2662	3.29410	.01324
7	.2875	.28748	3.8106	3.84312	.01545
8	.3285	.32855	4.3550	4 39213	.01765
9	.3696	.36962	4.8994	4.94115	.01986

* For the moon increase this by 180°.

TABLE NO. IV—CONTINUED.

Minutes.	Sun's Lon. and Anom.	Moon's Anom.	Moon's Longitude.	Lon. of Node.	Seconds.	Sun's Lon. and Anom.	Moon's Lon. and Anom.
1	.00068	.0091	.00915	.00004	1	.00001	.00015
2	.00137	.0181	.01830	.00007	2	.00002	.00030
3	.00205	.0272	.02745	.00011	3	.00003	.00046
4	.00274	.0363	.03660	.00015	4	.00005	.00061
5	.00342	.0454	.04575	.00018	5	.00006	.00076
6	.00411	.0544	.05490	.00022	6	.00007	.00091
7	.00479	.0635	.06405	.00026	7	.00008	.00107
8	.00548	.0726	.07320	.00029	8	.00009	.00122
9	.00616	.0817	.08235	.00033	9	.00010	.00137

TABLE V.

Days of the year reckoned from March.

Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Jan.	Feb.
1	32	62	93	123	154	185	215	246	276	307	338
2	33	63	94	124	155	186	216	247	277	308	339
3	34	64	95	125	156	187	217	248	278	309	340
4	35	65	96	126	157	188	218	249	279	310	341
5	36	66	97	127	158	189	219	250	280	311	342
6	37	67	98	128	159	190	220	251	281	312	343
7	38	68	99	129	160	191	221	252	282	313	344
8	39	69	100	130	161	192	222	253	283	314	345
9	40	70	101	131	162	193	223	254	284	315	346
10	41	71	102	132	163	194	224	255	285	316	347
11	42	72	103	133	164	195	225	256	286	317	348
12	43	73	104	134	165	196	226	257	287	318	349
13	44	74	105	135	166	197	227	258	288	319	350
14	45	75	106	136	167	198	228	259	289	320	351
15	46	76	107	137	168	199	229	260	290	321	352
16	47	77	108	138	169	200	230	261	291	322	353
17	48	78	109	139	170	201	231	262	292	323	354
18	49	79	110	140	171	202	232	263	293	324	355
19	50	80	111	141	172	203	233	264	294	325	356
20	51	81	112	142	173	204	234	265	295	326	357
21	52	82	113	143	174	205	235	266	296	327	358
22	53	83	114	144	175	206	236	267	297	328	359
23	54	84	115	145	176	207	237	268	298	329	360
24	55	85	116	146	177	208	238	269	299	330	361
25	56	86	117	147	178	209	239	270	300	331	362
26	57	87	118	148	179	210	240	271	301	332	363
27	58	88	119	149	180	211	241	272	302	333	364
28	59	89	120	150	181	212	242	273	303	334	365
29	60	90	121	151	182	213	243	274	304	335	366
30	61	91	122	152	183	214	244	275	305	336	
31		92		153	184		245		306	337	

This Table shows the month and day to which any number of days in a year, reckoned from the 1st of March, corresponds.

TABLE VI.

Annual Equation of the Moon's Perigee.

ARGUMENT—Sun's Anomaly.

Arg.	0°	2°	4°	6°	8°	10°	Arg.
−							+
0	.000	.013	.026	.038	.051	.063	35
1	.063	.076	.088	.101	.113	.126	34
2	.126	·138	.149	.161	.172	.183	33
3	.183	.194	.205	.216	.226	.236	32
4	.236	.245	.255	.264	.273	.282	31
5	.282	.290	.298	.305	.312	.319	30
6	.319	.325	.331	.337	.342	.347	29
7	.347	.351	.355	.359	.362	.365	28
8	.365	.367	.369	.370	.371	.371	27
9	.371	.371	.371	.370	.369	.367	26
10	.367	.365	.362	.359	.355	.351	25
11	.351	.347	.342	.337	.331	.324	24
12	.324	.318	.311	.304	.296	.288	23
13	.288	.280	.271	.261	.252	.242	22
14	.242	.231	.221	.210	.199	.188	21
15	.188	.177	.165	.153	.141	.129	20
16	.129	.117	.104	.092	.079	.066	19
17	.066	.052	.039	.026	.013	.000	18
Arg.	10°	8°	6°	4°	2°	0°	Arg.

The sun's attraction causes a progressive motion in the line of the moon's apsides, which affects the place of the perigee; and since the distance of the sun varies in different seasons of the year, its attraction also varies. This causes the motion to be more rapid at some times than at others, occasioning inequalities in the moon's anomaly, for which this Table furnishes the correction.

TABLE VII.

Annual Equation of the Moon's Node.

ARGUMENT—Sun's Anomaly.

Arg.	0°	2°	4°	6°	8°	10°	Arg.
−							+
0	.0000	.0053	.0106	.0159	.0212	.0264	35
1	.0264	.0316	.0367	.0418	.0469	.0520	34
2	.0520	.0570	.0619	.0667	.0714	.0760	33
3	.0760	.0805	.0849	.0892	.0934	.0975	32
4	.0975	.1015	.1053	.1090	.1126	.1160	31
5	.1160	.1192	.1223	.1253	.1281	.1308	30
6	.1308	.1333	.1356	.1378	.1398	.1417	29
7	.1417	.1434	.1449	.1462	.1373	.1481	28
8	.1481	.1488	.1493	.1496	.1499	.1500	27
9	.1500	.1498	.1494	.1488	.1481	.1473	26
10	.1473	.1463	.1451	.1437	.1421	.1402	25
11	.1402	.1381	.1359	.1336	.1313	.1289	24
12	.1289	.1263	.1235	.1205	.1273	.1138	23
13	.1138	.1103	.1067	.1030	.0992	.0953	22
14	.0953	.0913	.0871	.0828	.0784	.0740	21
15	.0740	.0695	.0648	.0600	.0551	.0501	20
16	.0501	.0452	.0403	.0355	.0306	.0257	19
17	.0257	.0206	.0155	.0104	.0052	.0000	18
Arg.	10°	8°	6°	4°	2°	0°	Arg.

The retrograde motion of the moon's nodes is caused by the sun's attraction, and as the distance of that luminary varies in different seasons of the year, its attraction must also vary, producing inequalities in the motion of the nodes, for which this Table supplies the correction.

TABLE VIII.

Equation of the Sun's Centre.

ARGUMENT—Sun's Anomaly.

Arg.	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	Arg.
+												−
0	.0000	.0343	.0685	.1028	.1370	.1711	.2052	.2393	.2733	.3071	.3409	35
1	.3409	.3746	.4081	.4415	.4748	.5079	.5408	.5736	.6062	.6386	.6708	34
2	.6708	.7028	.7345	.7660	.7972	.8282	.8590	.8895	.9196	.9495	.9790	33
3	.9790	1.0083	1.0372	1.0658	1.0941	1.1220	1.1493	1.1767	1.2035	1.2299	1.2559	32
4	1.2559	1.2815	1.3067	1.3315	1.3559	1.3799	1.4034	1.4264	1.4491	1.4711	1.4928	31
5	1.4928	1.5140	1.5347	1.5549	1.5747	1.5939	1.6127	1.6309	1.6486	1.6658	1.6824	30
6	1.6824	1.6986	1.7142	1.7293	1.7439	1.7578	1.7712	1.7841	1.7965	1.8112	1.8194	29
7	1.8194	1.8301	1.8401	1.8496	1.8585	1.8669	1.8747	1.8819	1.8885	1.8945	1.9000	28
8	1.9000	1.9049	1.9091	1.9128	1.9159	1.9185	1.9204	1.9217	1.9225	1.9226	1.9223	27
9	1.9223	1.9213	1.9197	1.9175	1.9148	1.9115	1.9075	1.9030	1.8980	1.8924	1.8862	26
10	1.8862	1.8794	1.8721	1.8642	1.8557	1.8467	1.8372	1.8270	1.8164	1.8052	1.7935	25
11	1.7935	1.7812	1.7684	1.7551	1.7412	1.7269	1.7120	1.6966	1.6807	1.6644	1.6475	24
12	1.6475	1.6301	1.6123	1.5940	1.5752	1.5560	1.5362	1.5161	1.4955	1.4745	1.4530	23
13	1.4530	1.4311	1.4088	1.3861	1.3630	1.3395	1.3155	1.2915	1.2666	1.2415	1.2162	22
14	1.2162	1.1904	1.1643	1.1379	1.1111	1.0840	1.0566	1.0289	1.0009	.9726	.9441	21
15	.9441	.9152	.8861	.8567	.8271	.7973	.7672	.7369	.7064	.6757	.6448	20
16	.6448	.6137	.5825	.5510	.5194	.4877	.4558	.4238	.3917	.3594	.3271	19
17	.3271	.2946	.2621	.2296	.1969	.1641	.1314	.0985	.0657	.0329	.0000	18
Arg.	10°	9°	8°	7°	6°	5°	4°	3°	2°	1°	0°	Arg.

Owing to the elliptical form of the sun's apparent orbit, it does not revolve with uniform angular velocity, and this Table shows the correction in its longitude, required to be made on that account.

The epoch of this Table is 1840.

TABLE IX.

Equation of the Moon's Centre.

ARGUMENT—Moon's Anomaly.

Arg.	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	Arg.
+												−
0	0.0000	0.1181	0.2362	0.3542	0.4721	0.5897	0.7072	0.8244	0.9412	1.0578	1.1738	35
1	1.1738	1.2896	1.4048	1.5223	1.6336	1.7471	1.8600	1.9721	2.0835	2.1944	2.3041	34
2	2.3041	2.4131	2.5212	2.6284	2.7346	2.8398	2.9439	3.0470	3.1490	3.2498	3.3494	33
3	3.3494	3.4478	3.5450	3.6409	3.7354	3.8287	3.9205	4.0109	4.0999	4.1874	4.2735	32
4	4.2735	4.3580	4.4410	4.5223	4.6022	4.6804	4.7569	4.8318	4.9050	4.9765	5.0463	31
5	5.0463	5.1143	5.1806	5.2451	5.3078	5.3687	5.4277	5.4849	5.5403	5.5938	5.6454	30
6	5.6454	5.6952	5.7480	5.7889	5.8330	5.8750	5.9152	5.9534	5.9897	6.0240	6.0564	29
7	6.0564	6.0868	6.1152	6.1417	6.1663	6.1889	6.2094	6.2281	6.2448	6.2595	6.2722	28
8	6.2722	6.2830	6.2919	6.2988	6.3038	6.3068	6.3079	6.3070	6.3043	6.2997	6.2931	27
9	6.2931	6.2847	6.2743	6.2621	6.2481	6.2322	6.2144	6.1948	6.1734	6.1502	6.1252	26
10	6.1252	6.0984	6.0699	6.0396	6.0076	5.9739	5.9384	5.9013	5.8624	5.8220	5.7799	25
11	5.7799	5.7362	5.6908	5.6439	5.5954	5.5453	5.4937	5.4406	5.3860	5.3300	5.2723	24
12	5.2723	5.2135	5.1531	5.0913	5.0281	4.9636	4.8978	4.8306	4.7622	4.6924	4.6215	23
13	4.6215	4.5493	4.4759	4.4013	4.3256	4.2487	4.1707	4.0915	4.0114	3.9302	3.8480	22
14	3.8480	3.7620	3.6806	3.5954	3.5093	3.4223	3.3344	3.2457	3.1562	3.0658	2.9747	21
15	2.9747	2.8828	2.7901	2.6968	2.6028	2.5081	2.4127	2.3168	2.2203	2.1232	2.0256	20
16	2.0256	1.9275	1.8289	1.7298	1.6303	1.5304	1.4301	1.3294	1.2285	1.1272	1.0256	19
17	1.0256	0.9238	0.8217	0.7194	0.6170	0.5144	0.4117	0.3088	0.2059	0.1030	0.0000	18
Arg.	10°	9°	8°	7°	6°	5°	4°	3°	2°	1°	0°	Arg.

Owing to the elliptical form of the moon's orbit, it does not revolve with uniform angular velocity, and this Table shows the correction in its longitude required to be made on that account.

TABLE X.

Annual Equation of the Moon's Longitude.

ARGUMENT—Sun's Mean Anomaly.

Arg.	0°	2°	4°	6°	8°	10°	Arg.
−							+
0	.0000	.0064	.0128	.0192	.0255	.0318	35
1	.0318	.0381	.0443	.0505	.0567	.0627	34
2	.0627	.0687	.0747	.0805	.0863	.0920	33
3	.0920	.0974	.1028	.1081	.1132	.1183	32
4	.1183	.1232	.1280	.1326	.1370	.1413	31
5	.1413	.1454	.1494	.1532	.1568	.1602	30
6	.1602	.1634	.1664	.1692	.1719	.1743	29
7	.1743	.1764	.1785	.1803	.1818	.1832	28
8	.1832	.1843	.1852	.1859	.1864	.1866	27
9	.1866	.1865	.1864	.1860	.1853	.1843	26
10	.1843	.1832	.1819	.1802	.1784	.1764	25
11	.1764	.1742	.1717	.1690	.1662	.1630	24
12	.1630	.1597	.1562	.1525	.1486	.1446	23
13	.1446	.1404	.1359	.1313	.1265	.1216	22
14	.1216	.1165	.1113	.1059	.1004	.0947	21
15	.0947	.0890	.0831	.0771	.0711	.0649	20
16	.0649	.0586	.0523	.0459	.0395	.0330	19
17	.0330	.0264	.0198	.0132	.0066	.0000	18
Arg.	10°	8°	6°	4°	2°	0°	Arg.

The attraction of the sun upon the moon tends to draw it away from the earth and thus dilate its orbit; and since the distance of the sun from the earth varies in different seasons of the year, its attraction also varies. This causes a variation in the size of the moon's orbit, as well as in its velocity, producing inequalities in its longitude, for which this Table gives the required corrections.

TABLE XI.

Secular Equation, showing the Acceleration of the Moon's Mean Motion.

ARGUMENT—The date.

Arg.	0	5
181	00	01
182	01	02
183	03	04
184	05	06
185	07	09
186	11	13
187	15	17
188	19	21
189	24	27
190	30	33

The eccentricity of the earth's orbit is, and has been for ages, slowly diminishing, which renders the sun's disturbing influence on the moon less and less every year, thus allowing the orbit of the latter to contract, diminishing its periodic time. This Table contains the necessary corrections from this cause, at intervals of five years during the present century.

NOTE.—The numbers in this Table are the 3d and 4th places of decimals of a degree; therefore in using them, two cyphers must be prefixed as decimals. Thus for 33 read .0033.

TABLE XII.

Variation.

ARGUMENT—Moon's Longitude diminished by that of the Sun.

Arg.	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	Arg.	Reverse the Signs.
0	+.0000	+.0204	+.0407	+.0610	+.0812	+.1013	+.1212	+.1410	+.1607	+.1801	+.1993	35	
1	+.1993	+.2183	+.2370	+.2553	+.2734	+.2911	+.3084	+.3254	+.3419	+.3580	+.3736	34	
2	+.3736	+.3887	+.4034	+.4175	+.4312	+.4443	+.4568	+.4687	+.4801	+.4908	+.5009	33	
3	+.5009	+.5104	+.5192	+.5274	+.5349	+.5418	+.5479	+.5534	+.5581	+.5622	+.5656	32	
4	+.5656	+.5682	+.5701	+.5713	+.5718	+.5716	+.5706	+.5689	+.5666	+.5634	+.5597	31	
5	+.5597	+.5551	+.5499	+.5440	+.5374	+.5302	+.5222	+.5137	+.5044	+.4946	+.4841	30	
6	+.4841	+.4730	+.4613	+.4490	+.4361	+.4227	+.4088	+.3944	+.3794	+.3640	+.3481	29	
7	+.3481	+.3317	+.3149	+.2978	+.2802	+.2623	+.2440	+.2251	+.2065	+.1874	+.1680	28	
8	+.1680	+.1483	+.1284	+.1084	+.0882	+.0679	+.0475	+.0270	+.0054	−.0142	−.0348	27	
9	−.0348	−.0554	−.0760	−.0965	−.1170	−.1373	−.1575	−.1775	−.1973	−.2170	−.2364	26	
10	−.2364	−.2555	−.2744	−.2929	−.3112	−.3291	−.3466	−.3638	−.3805	−.3969	−.4127	25	
11	−.4127	−.4281	−.4430	−.4575	−.4714	−.4847	−.4975	−.5097	−.5214	−.5324	−.5429	24	
12	−.5429	−.5526	−.5618	−.5703	−.5781	−.5853	−.5918	−.5976	−.6026	−.6070	−.6107	23	
13	−.6107	−.6137	−.6159	−.6174	−.6182	−.6183	−.6176	−.6162	−.6140	−.6111	−.6076	22	
14	−.6076	−.6033	−.5982	−.5925	−.5860	−.5789	−.5711	−.5626	−.5534	−.5435	−.5330	21	
15	−.5330	−.5220	−.5101	−.4977	−.4847	−.4712	−.4571	−.4424	−.4271	−.4114	−.3952	20	
16	−.3952	−.3785	−.3614	−.3437	−.3257	−.3073	−.2885	−.2694	−.2500	−.2302	−.2102	19	
17	−.2102	−.1899	−.1694	−.1487	−.1278	−.1067	−.0855	−.0642	−.0429	−.0215	−.0000	18	
Arg.	10°	9°	8°	7°	6°	5°	4°	3°	2°	1°	0°	Arg.	

Reverse the Signs.

Owing to the disturbing influence of the sun, the moon is alternately accelerated and retarded in the different quadrants, reckoning from syzygy. This Table contains the corrections in its longitude resulting from this cause.

TABLE XIII.

Annual Equation of Variation.

ARGUMENTS—The argument of Variation at the top and bottom, and the Sun's Mean Anomaly at the sides.

		90 270	95 275	100 280	105 285	110 290	115 295	120 300	125 305	130 310	135 315		
Arg.	Arg.	0 180	5 185	10 190	15 195	20 200	25 205	30 210	35 215	40 220	45 225		
0	180	.0000	−.0064	−.0125	−.0183	−.0233	−.0281	−.0314	−.0342	−.0358	−.0364	180	360
10	190	+.0008	−.0053	−.0114	−.0172	−.0225	−.0267	−.0306	−.0333	−.0353	−.0358	170	350
20	200	+.0019	−.0042	−.0100	−.0156	−.0211	−.0250	−.0286	−.0317	−.0333	−.0342	160	340
30	210	+.0028	−.0028	−.0083	−.0133	−.0181	−.0225	−.0261	−.0286	−.0306	−.0314	150	330
40	220	+.0033	−.0014	−.0061	−.0108	−.0153	−.0192	−.0225	−.0250	−.0267	−.0281	140	320
50	230	+.0042	+.0000	−.0039	−.0081	−.0119	−.0153	−.0181	−.0206	−.0225	−.0233	130	310
60	240	+.0047	+.0017	−.0019	−.0050	−.0081	−.0108	−.0133	−.0156	−.0172	−.0184	120	300
70	250	+.0050	+.0028	+.0006	−.0019	−.0039	−.0061	−.0083	−.0100	−.0114	−.0125	110	290
80	260	+.0053	+.0042	+.0028	+.0017	+.0000	−.0014	−.0028	−.0042	−.0053	−.0064	100	280
90	270	+.0053	+.0053	+.0053	+.0047	+.0042	+.0033	+.0028	+.0019	+.0008	.0000	90	270
100	280	+.0053	+.0064	+.0072	+.0078	+.0081	+.0083	+.0081	+.0078	+.0072	+.0064	80	260
110	290	+.0050	+.0072	+.0089	+.0106	+.0119	+.0128	+.0133	+.0133	+.0133	+.0125	70	250
120	300	+.0047	+.0078	+.0108	+.0131	+.0153	+.0169	+.0181	+.0189	+.0186	+.0184	60	240
130	310	+.0042	+.0081	+.0119	+.0153	+.0181	+.0206	+.0206	+.0233	+.0239	+.0233	50	230
140	320	+.0033	+.0083	+.0128	+.0169	+.0206	+.0236	+.0225	+.0275	+.0281	+.0281	40	220
150	330	+.0028	+.0084	+.0133	+.0181	+.0225	+.0258	+.0286	+.0306	+.0317	+.0314	30	210
160	340	+.0019	+.0078	+.0133	+.0189	+.0233	+.0275	+.0306	+.0328	+.0339	+.0342	20	200
170	350	+.0008	+.0072	+.0133	+.0186	+.0239	+.0281	+.0317	+.0339	+.0353	+.0358	10	190
		90 270	85 265	80 260	75 255	70 250	65 245	60 240	55 235	50 230	45 225	Arg.	Arg.
		180 360	175 355	170 350	165 345	160 340	155 335	150 330	145 325	140 320	135 315		

The inequality in the moon's motion, denominated Variation, and for which Table 12th furnishes the correction, being occasioned by the disturbing influence of the sun, must be greater or less according as the distance of the earth from that luminary varies. In that Table the earth is supposed to be at its *mean* distance; hence another correction becomes necessary, which this Table furnishes.

TABLE XIV.

Evection.

ARGUMENT—The Moon's Mean Anomaly diminished by twice the excess of the Moon's Mean Longitude over the True Longitude of the Sun.

Arg.	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	Arg.
–												+
0	.0000	.0238	.0475	.0713	.0950	.1187	.1423	.1659	.1894	.2129	.2363	35
1	.2363	.2596	.2829	.3061	.3291	.3521	.3750	.3978	.4204	.4428	.4651	34
2	.4651	.4873	.5093	.5312	.5530	.5744	.5958	.6170	.6379	.6587	.6793	33
3	.6793	.6996	.7197	.7396	.7593	.7787	.7979	.8168	.8355	.8539	.8720	32
4	.8720	.8899	.9074	.9247	.9417	.9584	.9748	.9909	1.0067	1.0222	1.0374	31
5	1.0374	1.0522	1.0667	1.0808	1.0947	1.1082	1.1213	1.1341	1.1465	1.1586	1.1703	30
6	1.1703	1.1817	1.1926	1.2032	1.2135	1.2234	1.2323	1.2420	1.2507	1.2590	1.2670	29
7	1.2670	1.2745	1.2816	1.2884	1.2948	1.3007	1.3063	1.3114	1.3162	1.3206	1.3245	28
8	1.3245	1.3281	1.3312	1.3339	1.3362	1.3381	1.3396	1.3407	1.3414	1.3417	1.3415	27
9	1.3415	1.3410	1.3400	1.3386	1.3369	1.3347	1.3321	1.3291	1.3257	1.3220	1.3178	26
10	1.3178	1 3132	1.3082	1.3028	1.2971	1.2909	1.2844	1.2774	1.2701	1.2624	1.2543	25
11	1.2543	1.2459	1.2370	1.2278	1.2182	1.2083	1.1980	1.1872	1.1763	1.1650	1.1533	24
12	1.1533	1.1412	1.1288	1.1161	1.1031	1.0897	1.0760	1.0619	1.0476	1.0330	1.0180	23
13	1.0180	1.0027	.9872	.9713	.9552	.9388	.9221	.9051	.8879	.8704	.8526	22
14	.8526	.8346	.8164	.7979	.7792	.7603	.7411	.7217	.7021	.6822	.6622	21
15	.6622	.6420	.6217	.6011	.5803	.5594	.5383	.5171	.4957	.4742	.4525	20
16	.4525	.4307	.4087	.3867	.3646	.3423	.3199	.2975	.2749	.2523	.2296	19
17	.2296	.2068	.1840	.1611	.1382	.1152	.0922	.0692	.0462	.0231	.0000	18
Arg.	10°	9°	8°	7°	6°	5°	4°	3°	2°	1°	0°	Arg.

The disturbing influence of the sun causes a variation in the eccentricity of the moon's orbit, and in the position of the major axis. This affects the moon's longitude, producing inequalities, for which this Table gives the required corrections.

TABLE XV.

Annual Equation of Evection.

ARGUMENTS—The argument for Evection at the top and bottom, and the Sun's Mean Anomaly at the sides.

		0	10	20	30	40	50	60	70	80	90		
Arg.	Arg.	180	190	200	210	220	230	240	250	260	270		
0	180	.0000	–.0081	–.0161	–.0236	–.0303	–.0364	–.0408	–.0444	–.0467	–.0472	360	180
10	190	+.0036	–.0047	–.0122	–.0200	–.0272	–.0331	–.0386	–.0425	–.0453	–.0467	350	170
20	200	+.0072	– 0005	– 0083	–.0158	–.0228	–.0294	–.0347	–.0394	–.0425	–.0444	340	160
30	210	+.0108	+.0033	–.0042	–.0111	–.0181	–.0244	–.0303	–.0347	–.0386	–.0408	330	150
40	220	+.0136	+.0072	+.0005	–.0064	–.0128	–.0189	–.0244	–.0294	–.0331	–.0364	320	140
50	230	+.0164	+.0108	+.0050	–.0011	–.0072	–.0128	–.0181	–.0228	–.0272	–.0303	310	130
60	240	+.0186	+.0142	+.0092	+.0042	–.0011	–.0064	–.0111	–.0158	–.0200	–.0236	300	120
70	250	+.0200	+.0175	+.0133	+.0092	+.0050	+.0005	–.0042	–.0083	–.0122	–.0161	290	110
80	260	+.0211	+.0192	+.0175	+.0142	+.0108	+.0072	+.0033	–.0005	–.0047	–.0081	280	100
90	270	+.0211	+.0211	+.0200	+.0186	+.0164	+.0136	+.0108	+.0072	+ 0036	.0000	270	90
100	280	+.0211	+.0219	+.0228	+.0222	+.0214	+.0200	+.0175	+.0150	+.0117	+.0081	260	80
110	290	+.0200	+.0228	+.0242	+.0256	+.0258	+.0253	+.0241	+.0219	+.0195	+.0161	250	70
120	300	+.0186	+.0222	+.0256	+.0278	+.0294	+.0300	+.0297	+.0286	+.0264	+.0236	240	60
130	310	+.0164	+.0214	+.0258	+.0294	+.0320	+.0339	+.0344	+.0342	+.0328	+.0303	230	50
140	320	+.0136	+.0200	+.0253	+.0300	+.0339	+.0364	+.0383	+.0386	+.0381	+.0364	220	40
150	330	+.0108	+.0175	+.0241	+.0297	+.0344	+.0383	+.0405	+.0422	+.0422	+.0408	210	30
160	340	+.0072	+.0150	+.0219	+.0286	+.0342	+.0386	+.0422	+.0422	+.0450	+.0444	200	20
170	350	+.0036	+.0117	+.0195	+.0264	+.0328	+.0381	+.0422	+.0450	+.0464	+.0467	190	10
180	360	+.0000	+.0081	+.0161	+.0236	+.0303	+.0364	+.0408	+.0444	+.0467	+.0472	180	0
		180	170	160	150	140	130	120	110	100	90	Arg.	Arg.
		360	350	340	330	320	310	300	290	280	270		

The inequality in the moon's motion denominated Evection, and for which Table 14th furnishes the correction, being occasioned by the disturbing influence of the sun, must be greater or less according as the distance of the earth from that luminary varies. In that Table the earth is supposed to be at its *mean* distance; hence another correction becomes necessary, which this Table furnishes.

NOTE.—The foregoing explanation is true so far as it goes; but the values given in the Table are partially referrible to a variation in the argument for Evection occasioned by the Annual Equation of the Moon's Longitude. And we might on this principle add several other Tables, which should contain corrections required by the alteration of the arguments of previous ones; but this would not accord with the simplicity of our design.

TABLE XVI.

Nodal Equation of the Moon's Longitude.

ARGUMENT—The Sun's longitude diminished by that of the Moon's Node.

Arg.		0°	2°	4°	6°	8°	10°	Arg.	
−	−							+	+
0	18	.0000	.0012	.0024	.0036	.0048	.0059	17	35
1	19	.0059	.0071	.0082	.0092	.0102	.0112	16	34
2	20	.0112	.0121	.0129	.0137	.0144	.0150	15	33
3	21	.0150	.0156	.0161	.0165	.0169	.0171	14	32
4	22	.0171	.0173	.0174	.0174	.0173	.0171	13	31
5	23	.0171	.0169	.0165	.0161	.0156	.0150	12	30
6	24	.0150	.0144	.0137	.0129	.0121	.0112	11	29
7	25	.0112	.0102	.0092	.0082	.0071	.0059	10	28
8	26	.0059	.0048	.0036	.0024	.0012	.0000	9	27
Arg.		10°	8°	6°	4°	2°	0°	Arg.	

The influence of the sun tending to dilate the moon's orbit is greatest when the former is in the plane of the latter; i. e. when passing the nodes, and least when farthest from them, occasioning an inequality in the moon's motion, for which this Table furnishes the necessary correction.

TABLE XVII.

Reduction.

ARGUMENT—The Longitude of the Moon, diminished by that of its Node.

Arg.		0°	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	Arg.	
−	−												+	+
0	18	.0000	.0039	.0079	.0118	.0157	.0196	.0235	.0273	.0311	.0349	.0386	17	35
1	19	.0386	.0423	.0460	.0496	.0531	.0565	.0599	.0632	.0664	.0696	.0726	16	34
2	20	.0726	.0756	.0785	.0813	.0837	.0866	.0891	.0914	.0937	.0958	.0979	15	33
3	21	.0979	.0998	.1016	.1033	.1048	.1062	.1075	.1087	.1096	.1105	.1113	14	32
4	22	.1113	.1119	.1124	.1127	.1129	.1130	.1129	.1127	.1124	.1119	.1113	13	31
5	23	.1113	.1105	.1096	.1087	.1075	.1062	.1048	.1033	.1016	.0998	.0979	12	30
6	24	.0979	.0958	.0937	.0914	.0891	.0866	.0837	.0813	.0785	.0756	.0726	11	29
7	25	.0726	.0696	.0664	.0632	.0599	.0565	.0531	.0496	.0460	.0423	.0386	10	28
8	26	.0386	.0349	.0311	0273	.0235	.0196	.0157	.0118	.0079	.0039	0000	9	27
Arg.		10°	9°	8°	7°	6°	5°	4°	3°	2°	1°	0°	Arg.	

The preceding Tables give the Moon's Longitude in its orbit: this reduces it to the Ecliptic.

TABLE XVIII.

Lunar or Menstrual Equation of the Sun's Longitude.

ARGUMENT—The Longitude of the Moon diminished by that of the Sun.

Arg.		0°	5°	10°	Arg.	
+	−				+	−
0	18	.0000	.0002	.0004	17	35
1	19	.0004	.0005	.0007	16	34
2	20	.0007	.0009	.0010	15	33
3	21	.0010	.0012	.0013	14	32
4	22	.0013	.0015	.0016	13	31
5	23	.0016	.0017	.0018	12	30
6	24	.0018	.0019	.0020	11	29
7	25	.0020	.0020	.0021	10	28
8	26	.0021	.0021	.0021	9	27
Arg.		10°	5°	0°	Arg.	

The revolution of the earth around the common centre of gravity of the moon and earth, affects the sun's apparent place, causing a change in its longitude for which this Table furnishes the correction.

TABLE XIX.

Lunar Nutation in Longitude.

ARGUMENT—Longitude of the Moon's Ascending Node.

Arg. −	Arg. −		Arg. +	Arg. +
0°	180°	.0000	180°	360°
10	170	.0009	190	350
20	160	.0017	200	340
30	150	.0024	210	330
40	140	.0031	220	320
50	130	.0037	230	310
60	120	.0042	240	300
70	110	.0046	250	290
80	100	.0047	260	280
90	90	.0048	270	270

The Precession of the Equinoxes, being occasioned in part by the attraction of the moon lying out of the plane of the earth's equator, must be more or less rapid according to the obliquity of the plane of its orbit to that of the equator, which depends on the longitude of the moon's nodes. The longitudes of all the heavenly bodies, being reckoned from the Vernal Equinox, must be affected by any change in the place of the point from which they are reckoned, and therefore need a correction from this cause, which this Table supplies.

TABLE XX.

Sun's Semi-diameter and Horary Motion.

ARGUMENT—Sun's Anomaly.

Arg.	Semi-diameter.	Horary motion.	Arg.
0°	.2717	.0425	360°
10	.2716	.0424	350
20	.2714	.0423	340
30	.2710	.0422	330
40	.2705	.0421	320
50	.2700	.0419	310
60	.2694	.0417	300
70	.2687	.0415	290
80	.2679	.0413	280
90	.2671	.0411	270
100	.2663	.0409	260
110	.2655	.0407	250
120	.2648	.0405	240
130	.2642	.0403	230
140	.2637	.0401	220
150	.2632	.0400	210
160	.2628	.0399	200
170	.2626	.0398	190
180	.2625	.0397	180

The elliptical shape of the sun's apparent orbit causes it to vary, both as to apparent size and velocity. The values of these are given in this Table, at intervals of 10° throughout the entire orbit.

TABLE XXI.

Moon's Semi-diameter, Horary Motion and Equatorial Parallax.

ARGUMENT—Moon's corrected Anomaly.

Argument.	Semi-diameter.	Horary motion.	Equat. Parallax.	Argument.
0°	.2743	.6133	1°.0051	360
5	.2742	.6131	1°.0049	355
10	.2740	.6122	1°.0042	350
15	.2737	.6106	1°.0030	345
20	.2733	.6086	1°.0014	340
25	.2727	.6061	.9993	335
30	.2719	.6031	.9968	330
35	.2712	.5994	.9939	325
40	.2703	.5956	.9907	320
45	.2693	.5911	.9871	315
50	.2682	.5864	.9833	310
55	.2671	.5816	.9792	305
60	.2659	.5767	.9749	300
65	.2647	.5714	.9705	295
70	.2635	.5661	.9659	290
75	.2623	.5608	.9613	285
80	.2611	.5556	.9567	280
85	.2599	.5503	.9521	275
90	.2586	.5450	.9475	270
95	.2574	.5397	.9429	265
100	.2562	.5350	.9384	260
105	.2550	.5303	.9342	255
110	.2539	.5256	.9302	250
115	.2528	.5214	.9263	245
120	.2518	.5173	.9227	240
125	.2509	.5136	.9194	235
130	.2500	.5100	.9162	230
135	.2492	.5069	.9133	225
140	.2485	.5039	.9108	220
145	.2479	.5014	.9086	215
150	.2474	.4992	.9066	210
155	.2469	.4972	.9048	205
160	.2465	.4958	.9034	200
165	.2462	.4944	.9023	195
170	.2460	.4936	.9016	190
175	.2459	.4932	.9011	185
180	.2458	.4930	.9009	180

The principle and construction of this Table is the same as that of Table 20th. At the time of new or full moon the quantities in this Table must be increased for the effect of Variation as follows, viz. 1st column, .0020; 2d do. .0115; 3d do. .0073.

TABLE XXII.

Moon's Semi-diameter, Hourly Motion, and Equatorial Parallax, as affected by Evection.

ARGUMENT—The same as for Evection, Table 14th.

Arg.	Semi-diameter.	Hourly motion.	Equatorial Parallax.	Arg.
0°	+.0029	+.0112	+.0105	360°
10	+.0028	+.0109	+.0103	350
20	+.0027	+.0103	+.0098	340
30	+.0025	+.0097	+.0091	330
40	+.0022	+.0086	+.0081	320
50	+.0019	+.0070	+.0068	310
60	+.0014	+.0055	+.0052	300
70	+.0009	+.0036	+.0035	290
80	+.0005	+.0019	+.0018	280
90	−.0000	−.0002	−.0001	270
100	−.0005	−.0020	−.0019	260
110	−.0010	−.0038	−.0037	250
120	−.0014	−.0055	−.0054	240
130	−.0619	−.0071	−.0068	230
140	−.0022	−.0083	−.0080	220
150	−.0024	−.0094	−.0089	210
160	−.0026	−.0103	−.0096	200
170	−.0027	−.0107	−.0100	190
180	−.0028	−.0109	−.0103	180

All the inequalities in the moon's longitude, for which the foregoing Tables give the corrections, must likewise affect its apparent size, hourly motion, and equatorial parallax. Variation and Evection are the only ones that it is important to take into account, the former of which may be considered constant at the time of new or full moon, and this Table gives the requisite correction for the latter.

TABLE XXIII.

Obliquity of the Ecliptic to the Equator.

ARGUMENT—The date.

Arg.	0	1	2	3	4	5	6	7	8	9
	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″
184	23 27 15.1	23 27 43.2	23 27 40.4	23 27 37.1	23 27 33.5	23 27 29.6	23 27 27.1	23 27 24.8	23 27 23.6	23 27 23.4
185	23 27 24.3	23 27 26.0	23 27 28.3	23 27 31.0	23 27 33.6	23 27 35.7	23 27 37.2	23 27 37.7	23 27 37.3	23 27 36.0
186	23 27 33.6	23 27 30.7	23 27 27.2	23 27 23.7	23 27 21.3	23 27 17.6	23 27 15.8	23 27 14.9	23 27 15.1	23 27 15.9
187	23 27 18.4	23 27 20.1	23 27 23.5	23 27 26.0	23 27 27.9	23 27 29.0	23 27 29.2	23 27 28.5	23 27 26 7	23 27 24.1
188	23 27 20.8	23 27 17.4	23 27 13.9	23 27 10.8	23 27 8.4	23 27 6.8	23 27 6.3	23 27 7.0	23 27 8.6	23 27 10.8
189	23 27 13.4	23 27 16.0	23 27 18.3	23 27 20.0	23 27 20.7	23 27 20 5	23 27 19.4	23 27 17.3	23 27 14.4	23 27 11.0

The obliquity of the ecliptic to the equator is slowly diminishing, owing to the attraction of the planets, and is also subject to an inequality whose period is about nineteen years, caused by the attraction of the moon, and called Nutation. This Table gives the obliquity on the 1st of January in each year, taking both these causes into account.

TABLE XXIV.

Moon's Latitude in Eclipses.

ARGUMENT—Moon's Longitude diminished by that of its Node.

Arg.		.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	1°.0	Arg.	
N.A.	S.D.												N.D.	S.A.
°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
0	180	0.0000	0.0087	0.0175	0.0262	0.0350	0.0437	0.0524	0.0611	0.0698	0.0785	0.0873	179	359
1	181	0.0873	0.0960	0.1048	0.1135	0.1222	0.1310	0.1397	0.1484	0.1572	0.1659	0.1746	178	358
2	182	0.1746	0.1834	0.1921	0.2008	0.2095	0.2182	0.2269	0.2356	0.2443	0.2530	0.2617	177	357
3	183	0.2617	0.2704	0.2791	0.2878	0.2965	0.3052	0.3139	0.3226	0.3313	0.3399	0.3486	176	356
4	184	0.3486	0.3572	0.3659	0.3746	0.3832	0.3919	0.4005	0.4092	0.4180	0.4267	0.4353	175	355
5	185	0.4353	0.4440	0.4526	0.4613	0.4699	0.4786	0.4873	0.4960	0.5046	0.5133	0.5219	174	354
6	186	0.5219	0.5306	0.5392	0.5478	0.5565	0.5652	0.5739	0.5826	0.5912	0.5999	0.6085	173	353
7	187	0.6085	0.6173	0.6258	0.6343	0.6431	0.6517	0.6604	0.6690	0.6778	0.6864	0.6950	172	352
8	188	0.6950	0.7036	0.7122	0.7208	0.7295	0.7381	0.7467	0.7553	0.7639	0.7725	0.7811	171	351
9	189	0.7811	0.7897	0.7983	0.8069	0.8155	0.8241	0.8327	0.8413	0.8499	0.8585	0.8672	170	350
10	190	0.8672	0.8758	0.8844	0.8930	0.9015	0.9101	0.9187	0.9273	0.9358	0.9444	0.9529	169	349
11	191	0.9529	0.9614	0.9700	0.9785	0.9870	0.9955	1.0041	1.0126	1.0211	1.0296	1.0382	168	348
12	192	1.0382	1.0467	1.0552	1.0638	1.0723	1.0808	1.0893	1.0978	1.1063	1.1148	1.1233	167	347
13	193	1.1233	1.1318	1.1403	1.1488	1.1573	1.1658	1.1743	1.1828	1.1913	1.1998	1.2082	166	346
14	194	1.2082	1.2167	1.2251	1.2335	1.2420	1.2504	1.2588	1.2672	1.2756	1.2841	1.2925	165	345
15	195	1.2925	1.3009	1.3093	1.3177	1.3261	1.3345	1.3428	1.3512	1.3596	1.3680	1.3765	164	344
16	196	1.3765	1.3849	1.3933	1.4017	1.4101	1.4185	1.4269	1.4353	1.4437	1.4521	1·4605	163	343
Arg.		1°.0	.9	.8	.7	.6	.5	.4	3.	.2	.1	.0	Arg.	

The moon has sometimes a north and sometimes a south latitude, owing to the obliquity of the plane of its orbit to that of the ecliptic. This Table gives the latitude for every tenth of a degree of longitude, reckoned 17° either way from each node. The capital letters at the head of the columns of the argument show whether the latitude is north or south, and whether it is ascending or descending.

TABLE XXV.

Angle of the visible path of the Moon with the Ecliptic in Eclipses.

ARGUMENTS—Horary motion of the Moon from the Sun at the top, and the Moon's distance from the Node at the right and left.

N. A.	S. D.	.44	.46	.48	.50	.52	.54	.56	.58	.60	N. D.	S. A.
0°	180°	5°47′	5°46′	5°45′	5°44′	5°43′	5°42′	5°41′	5°40′	5°39′	180°	360°
3	183	5 46	5 45	5 44	5 43	5 42	5 41	5 40	5 40	5 39	177	357
6	186	5 45	5 44	5 43	5 42	5 41	5 40	5 39	5 39	5 38	174	354
9	189	5 42	5 41	5 40	5 39	5 38	5 38	5 37	5 36	5 35	171	351
12	192	5 39	5 38	5 37	5 36	5 35	5 34	5 34	5 33	5 32	168	348
15	195	5 35	5 34	5 33	5 32	5 31	5 30	5 30	5 29	5 28	165	345

The angle of the moon's path with the ecliptic, which depends upon its distance from the node, is apparently increased by the earth's motion in the same direction. This Table gives the apparent angle, taking both these facts into consideration.

TABLE XXVI.

The Sun's Declination.

Argument—Sun's Longitude.

Arg.		0°	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	Arg.	
N.	S.	°	°	°	°	°	°	°	°	°	°	°	S.	N.
0	18	0.0000	0.3980	0.7961	1.1939	1.5911	1.9883	2.3850	2.7809	3.1761	3.5703	3.9639	35	17
1	19	3.9639	4.3564	4.7477	5.1380	5.5267	5.9142	6.3000	6.6839	7.0664	7.4472	7.8259	34	16
2	20	7.8259	8.2025	8.5767	8.9489	9.3189	9.6858	10.0506	10.4128	10.7720	11.1284	11.4817	33	15
3	21	11.4817	11.8317	12.1789	12.5225	12.8628	13.1997	13.5328	13.8622	14.1877	14.5092	14.8270	32	14
4	22	14.8270	15.1402	15.4494	15.7539	16.0542	16.3500	16.6408	16.9273	17.2086	17.4850	17.7561	31	13
5	23	17.7561	18.0223	18.2833	18.5386	18.7886	19.0331	19.2717	19.5042	19.7314	19.9528	20.1681	30	12
6	24	20.1681	20.3770	20.5795	20.7761	20.9664	21.1500	21.3267	21.4975	21.6614	21.8186	21.9689	29	11
7	25	21.9689	22 1122	22.2486	22.3781	22.5003	22.6153	22.7234	22.8242	22.9178	23.0042	23.0831	28	10
8	26	23.0831	23.1544	23.2184	23.2750	23.3242	23.3658	23.3997	23.4261	23.4450	23.4564	23.4603	27	9
Arg.		10°	9°	8°	7°	6°	5°	4°	3°	2°	1°	0°	Arg.	

The plane of the ecliptic not coinciding with that of the equator, the sun is sometimes north of the equator and sometimes south. This is called its declination, and this Table shows its amount for every degree of longitude. The epoch of the Table is 1840.

TABLE XXVII.

1st *Preliminary Equation.*

Argument—Moon's Anomaly.

Arg.	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	Arg.
−	h. m. s.	h. m. s.	h. m. s.	h. m. s.	h. m. s.	h. m. s.	h. m. s.	h. m. s.	h. m. s.	h. m. s.	h. m. s.	+
0	0 0 0	0 9 34	0 19 8	0 28 41	0 38 13	0 47 44	0 57 13	1 6 41	1 16 7	1 25 31	1 34 54	35
1	1 34 54	1 44 16	1 53 36	2 2 53	2 12 8	2 21 19	2 30 28	2 39 34	2 48 39	2 57 43	3 6 45	34
2	3 6 45	3 15 44	3 24 42	3 33 38	3 42 32	3 1 23	4 0 7	4 8 47	4 17 25	4 26 1	4 34 33	33
3	4 34 33	4 43 2	4 51 15	4 59 42	5 7 56	5 16 5	5 24 9	5 32 9	5 40 4	5 47 54	5 55 38	32
4	5 55 38	6 3 16	6 10 49	6 18 18	6 25 40	6 32 56	6 40 6	6 47 6	6 54 8	7 1 2	7 7 50	31
5	7 7 50	7 14 30	7 21 2	7 27 22	7 33 36	7 39 46	7 45 46	7 51 33	7 57 23	8 3 12	8 8 59	30
6	8 8 59	8 14 33	8 20 18	8 25 44	8 31 0	8 36 6	8 41 2	8 45 48	8 50 24	8 54 50	8 58 6	29
7	8 58 6	9 3 13	9 7 9	9 10 54	9 14 28	9 17 51	9 21 3	9 24 4	9 26 54	9 29 33	9 32 1	28
8	9 32 1	9 34 18	9 36 24	9 38 19	9 40 3	9 41 36	9 42 59	9 44 11	9 45 12	9 46 3	9 46 44	27
9	9 46 44	9 47 14	9 47 33	9 47 46	9 47 54	9 47 49	9 47 36	9 47 13	9 46 38	9 45 52	9 44 53	26
10	9 44 53	9 43 42	9 42 21	9 40 51	9 39 8	9 37 14	9 35 12	9 32 58	9 30 32	9 27 58	9 25 12	25
11	9 25 12	9 22 14	9 19 5	9 15 43	9 12 9	9 8 25	9 4 31	9 0 25	8 56 10	8 51 45	8 47 8	24
12	8 47 8	8 42 18	8 37 19	8 32 11	8 26 53	8 21 24	8 15 46	8 9 57	8 3 56	7 57 45	7 51 24	23
13	7 51 24	7 44 51	7 38 9	7 31 18	7 24 10	7 17 9	7 9 52	7 2 24	6 54 46	6 47 0	6 39 4	22
14	6 39 4	6 30 57	6 22 41	6 14 19	6 5 51	5 57 17	5 48 37	5 39 51	5 30 57	5 21 56	5 12 48	21
15	5 12 48	5 3 33	4 54 11	4 44 42	4 35 6	4 25 20	4 15 26	4 5 26	3 55 21	3 45 11	3 34 58	20
16	3 34 58	3 24 42	3 14 24	3 4 3	2 53 38	2 43 9	2 32 34	2 21 54	2 11 10	2 0 23	1 49 33	19
17	1 49 33	1 38 40	1 27 44	1 16 46	1 5 48	0 54 50	0 43 52	0 32 54	0 21 56	0 10 58	0 0 0	18
Arg.	10°	9°	8°	7°	6°	5°	4°	3°	2°	1°	0°	Arg.

When the moon's anomaly is less than 180°, it is in advance of its mean place at time of new or full moon by reason of the Equation of the Centre, but behind it by Evection, (Tables 9 and 14,) yet on the whole it is in advance; consequently it will overtake the sun sooner than it would otherwise do, and something must be subtracted from the mean time. The contrary takes place when the anomaly is more than 180°. This Table shows the amount of time to be added or subtracted from these causes.

TABLE XXVIII.

2d *Preliminary Equation.*

ARGUMENT—Sun's Anomaly.

Arg. +	0° h. m. s.	1° h. m. s.	2° h. m. s.	3° h. m. s.	4° h. m. s.	5° h. m. s.	6° h. m. s.	7° h. m. s.	8° h. m. s.	9° h. m. s.	10° h. m. s.	Arg. −
0	0 0 0	0 4 29	0 8 56	0 13 23	0 17 50	0 22 17	0 26 44	0 31 10	0 35 36	0 40 2	0 44 28	35
1	0 44 28	0 48 52	0 53 13	0 57 36	1 1 56	1 6 15	1 10 13	1 14 49	1 19 5	1 23 19	1 27 31	34
2	1 27 31	1 31 41	1 35 49	1 39 56	1 41 1	1 48 4	1 52 6	1 56 5	2 0 1	2 3 55	2 7 45	33
3	2 7 45	2 11 35	2 15 20	2 19 5	2 22 47	2 26 26	2 30 2	2 33 35	2 37 6	2 40 33	2 43 57	32
4	2 43 57	2 47 18	2 50 36	2 53 49	2 57 0	3 0 7	3 3 10	3 6 10	3 9 6	3 11 59	3 14 49	31
5	3 14 49	3 17 35	3 20 20	3 23 0	3 25 35	3 28 5	3 30 30	3 32 50	3 35 6	3 37 19	3 39 30	30
6	3 39 30	3 41 40	3 43 45	3 45 44	3 47 38	3 49 26	3 51 9	3 52 49	3 54 26	3 55 59	3 57 27	29
7	3 57 27	3 58 52	4 0 12	4 1 26	4 2 35	4 3 40	4 4 41	4 5 37	4 6 29	4 7 16	4 7 59	28
8	4 7 59	4 8 37	4 9 10	4 9 39	4 10 4	4 10 24	4 10 39	4 10 49	4 10 54	4 10 57	4 10 53	27
9	4 10 53	4 10 45	4 10 33	4 10 16	4 9 55	4 9 29	4 8 57	4 8 21	4 7 41	4 6 58	4 6 10	26
10	4 6 10	4 5 18	4 4 22	4 3 23	4 2 18	4 1 7	3 59 49	3 58 27	3 57 2	3 55 35	3 54 4	25
11	3 54 4	3 52 29	3 50 50	3 49 7	3 47 17	3 45 25	3 43 26	3 41 23	3 39 18	3 37 10	3 35 0	24
12	3 35 0	3 32 45	3 30 26	3 28 3	3 25 36	3 23 5	3 20 30	3 17 51	3 15 9	3 12 24	3 9 36	23
13	3 9 36	3 6 45	3 3 51	3 0 54	2 57 53	2 54 48	2 51 40	2 48 30	2 45 18	2 42 3	2 38 44	22
14	2 38 44	2 35 22	2 31 57	2 28 29	2 25 9	2 21 27	2 17 52	2 14 14	2 10 36	2 6 55	2 3 12	21
15	2 3 12	1 59 26	1 55 37	1 51 46	1 47 54	1 44 1	1 40 6	1 36 10	1 32 12	1 28 12	1 24 10	20
16	1 24 10	1 20 6	1 16 0	1 11 53	1 7 45	1 3 56	0 59 27	0 55 17	0 51 4	0 46 52	0 42 39	19
17	0 42 39	0 38 26	0 34 11	0 28 55	0 25 39	0 21 24	0 17 8	0 12 51	0 8 35	0 0 18	0 4 0	18
Arg.	10°	9°	8°	7°	6°	5°	4°	3°	2°	1°	0°	Arg.

When the sun's anomaly is less than 180°, it is before and the moon behind the mean place, by reason of the Equation of the Centre (Table 8) of the former, and the Annual Equation of the Longitude (Table 10) of the latter. For both reasons, then, the moon will not overtake the sun so soon as it would otherwise do, and consequently something must be added to the mean time of New or Full Moon. The contrary takes place when the anomaly is more than 180°; and this Table shows the amount of time to be added or subtracted from these causes.

TABLE XXIX.

Augmentation of the Moon's Semi-diameter.

ARGUMENT—Distance of the place (as projected on the disc) from the earth's centre.

Arg. +	
0	.0045
10	.0045
20	.0044
30	.0043
40	.0041
50	.0038
60	.0035
70	.0031
80	.0024
90	.0015
100	.0000

Tables 21 and 22 show us the apparent semi-diameter of the moon as viewed from the centre of the earth; but the distance of the moon from any place on the earth's surface at which it is visible (save when it is in the horizon) is less than from the centre, which must cause it to subtend a greater angle. This Table shows the augmentation in the moon's apparent semi-diameter from this cause.

TABLE XXX.

To convert minutes into decimals of a degree.

ARGUMENT—The number of minutes.

Arg.	0′	1′	2′	3′	4′	5′	6′	7′	8′	9′
0	.0000	.0167	.0333	.0500	.0667	.0833	.1000	.1167	.1333	.1500
1	.1667	.1833	.2000	.2167	.2333	.2500	.2667	.2833	.3000	.3167
2	.3333	.3500	.3667	.3833	.4000	.4167	.4333	.4500	.4667	.4833
3	.5000	.5167	.5333	.5500	.5667	.5833	.6000	.6167	.6333	.6500
4	.6667	.6833	.7000	.7167	.7333	.7500	.7667	.7833	.8000	.8167
5	.8333	.8500	.8667	.8833	.9000	.9167	.9333	.9500	.9667	.9833

TABLE XXXI.

To convert seconds into decimals of a degree.

ARGUMENT—The number of seconds.

Arg.	0″	1″	2″	3″	4″	5″	6″	7″	8″	9″
0	.0000	.0003	.0006	.0008	.0011	.0014	.0017	.0019	.0022	.0025
1	.0028	.0031	.0033	.0036	.0039	.0042	.0044	.0047	.0050	.0053
2	.0056	.0058	.0061	.0064	.0067	.0069	.0072	.0075	.0078	.0081
3	.0083	.0086	.0089	.0092	.0094	.0097	.0100	.0103	.0106	.0108
4	.0111	.0114	.0117	.0119	.0122	.0125	.0128	.0131	.0133	.0136
5	.0139	.0142	.0144	.0147	.0150	.0153	.0156	.0158	.0161	.0164